TEXTBOOK OF LOGIC

by the same author

A HISTORY OF SCIENCE, TECHNOLOGY AND PHILOSOPHY

THE CORRESPONDENCE OF SPINOZA

THE OLDEST BIOGRAPHY OF SPINOZA

THE PHILOSOPHY OF NEITZSCHE

ESSENTIALS OF LOGIC

EXERCISES IN LOGIC AND SCIENTIFIC METHOD

KEY TO EXERCISES IN LOGIC AND SCIENTIFIC METHOD

TEXTBOOK OF LOGIC

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SECOND EDITION
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"Nothing can be more important than the art of formal reasoning according to true Logic."—LEIBNIZ.

PREFACE TO THE SECOND EDITION

VARIOUS English and American teachers of Logic have kindly sent me spontaneous expressions of appreciation of this *Textbook*, and I gladly take this opportunity to thank them warmly. Some of them have also suggested a few additions in an Appendix, so as not to disturb the plan of the book. Although I am so fully occupied with the second volume of my *History of Science, Technology, and Philosophy* that I had to let this Textbook go out of print for a while, I have willingly made time in order to accede to the wishes of the teachers by adding a considerable Appendix. I have also made a few improvements in the body of the book; and I hope that its usefulness has been increased thereby.

February 1938

PREFACE

THERE are so many textbooks of logic that the publication of another one may require justification. My excuse is that this book is very different from the others, and likely to be more helpful to students and teachers alike. Logic is here treated consistently and adequately as the study of the main types of reasoning and the general conditions of their validity. All topics not strictly relevant to these problems are omitted. At the same time many types of reasoning not included in other textbooks are dealt with here. Thus the usual obstructions are removed, and the usual deficiencies are made good, so as to enable the reader to have a clearer and more comprehensive view of the whole field. Moreover, the exposition of the subject in this volume is more systematic and coherent than is usual. Strange though it may appear, textbooks of logic are among the chief sinners against the canons of logical method.

The time will come when the rudiments of logic will be taught in schools as well as in colleges, much as algebra and geometry are taught now. That the time has not arrived yet, and is not even in sight, is due mainly to the way in which logic is mostly taught. All too frequently the logic class is treated as the recruiting place for the sadly depleted philosophy classes. The attempt to interest them in philosophy by premature philosophical digressions only helps to bewilder most of the students. If mathematics were treated in that way, as it easily could be, then most undergraduates and many seniors would still be counting on their fingers or with the help of the abacus.

To those who are philosophically minded almost any

study will suggest philosophical problems sooner or later. Logic or mathematics or physics or astronomy, etc., may thus stimulate philosophical interest. But in the earlier stages it is best to treat them all as more or less self-contained studies, and to make sure that students master the rudiments of the subjects before attempting prematurely to deal with problems which may be beyond them even later on. Otherwise the result is deplorable. Most logic students know neither logic nor philosophy, when they might certainly have been taught some logic even if they have no aptitude for philosophy. And even some of those who have an aptitude for philosophy are hampered ever afterwards by an insufficient training in logic. Hence the urgent need for such a self-contained treatment of logic as is attempted in the present volume.

Lastly, a word about exercises. It cannot be emphasized too strongly that logic, like algebra and geometry, cannot be mastered by mere reading. The learner must work sufficient exercises to acquire facility in the analysis and criticism of actual arguments and investigations. Some material for this purpose will be found at the end of this volume. Additional material is contained in the author's *Exercises in Logic and Scientific Method*, and sufficient help with them is given in the *Key to the Exercises.*

November 1929

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INTRODUCTION

CHAPTER I

LOGIC AND SCIENTIFIC METHOD

§ 1. *The Scope of Logic.*

Logic is the study of the general conditions of valid inference (or of proof). To make this description intelligible it is necessary to explain some of its constituent terms, more especially the terms *inference* and *valid.*

Inference. An inference is an inferred judgment, that is, a judgment derived from another judgment, or from other judgments. All knowledge and all beliefs consist of judgments. In ordinary usage it is customary to distinguish between *knowledge* and *belief.* There are some things which we claim to *know,* there are others which we do not claim to know, but which we still *believe.* When used correctly the term *belief* is the more general term of the two, and includes *knowledge.* If we *believe* what we do *not* claim to *know,* we certainly believe what we *do* claim to know. In this sense, *knowledge* may be described as adequately justified belief, whereas beliefs not adequately justified may be described as *mere beliefs.* Used in the wide sense just explained, namely as including both *knowledge* and *mere beliefs,* the term *belief* is synonymous with *judgment,* as the term is used in Logic and Psychology. Now broadly speaking there are two kinds of judgments in respect of their origin, or felt origin. Some are obviously derived from other judgments, while others are not so derived. For example, I look at the sky at sunset and, seeing that it is red, I believe that it will be fine to-morrow. My belief that it will be fine to-morrow is derived from

(1) my observation of the red sky, and (2) my belief in the connection between a red sky at sunset and fine weather to follow. Such a derived belief or judgment is called an *inference*. On the other hand, my belief that the sky is red is not derived, but is the result of direct observation, while the belief in the connection between a red sky at sunset and fine weather to follow, may be either the immediate result of suggestion, or an inference derived from my belief in the credibility of my informant, or from my knowledge of the physical facts involved. Inferences, then, are *derived judgments*; judgments which are not derived from other judgments may be called *immediate* or *intuitive judgments*. Such immediate or intuitive judgments result either from perception by means of the senses (judgments of perception, or *perceptual judgments*), or from that kind of intellectual intuition to which we owe such self-evident truths as the axioms of geometry, etc. (*intuitive judgments* in the stricter sense). It is not always easy to distinguish an immediate from an inferential judgment. With the progress of knowledge and critical discrimination it is easily recognized that many judgments commonly regarded as immediate are really inferential. Still, the difficulty should not be exaggerated. The main point with which we are concerned at the moment is, that Logic, unlike Epistemology (or the Theory of Knowledge), is not concerned with all kinds of judgments, but only with those which are professedly derived from, or based upon, other judgments. Logic is the study of *inferences* not of beliefs generally.

The terms *inference* and *reasoning* are frequently used as synonyms. It is better, however, to distinguish between them. People frequently jump to conclu-

sions quite uncritically, without deliberation. At other times, in cases of special difficulty or importance, they proceed more cautiously, and carefully weigh the evidence for and against. It is best to confine the term *reasoning* to such deliberate reflection or *critical inference*. Strictly speaking, Logic is mainly concerned with *reasoning* in the sense just explained. Uncritical (or unreflective) inferences are as a rule neither controlled nor studied, except sometimes after the event. That is why they are frequently so mischievous.

A word may be added here about the relation of *inference* to *proof*, which terms appear almost as synonyms in the definition of Logic given above. In every argument there are two things: (1) the premises (or data, or evidence) and (2) the conclusion (or the inference). The conclusion is *inferred from* the evidence, and, if it is inferred accurately, the evidence is said to *prove* the conclusion. Thus *valid inference* and *proof* are simply different aspects of the same thing. If we start from the evidence (or premises) and proceed to the conclusion the process is called *inference*; but if we begin by entertaining some belief or suggestion and proceed to justify it by means of suitable evidence, the process is called *proof*. It is common, however, to make this further distinction between *inference* and *proof*. The term *proof* is usually restricted to *correct* proof, whereas the term inference is applied to incorrect as well as to correct inference. Thus, for example, inference from analogy is a very common type of inference (or of reasoning), but it is no *proof*, as will be explained in due course. Another point worth noting here is that the term *reasoning* is commonly applied to both (reflective) *inference* and *proof*, that is to say, reasoning may take the form of

drawing conclusions from evidence, or of finding evidence for beliefs or suggestions already entertained.

Validity. It was stated above that Logic is concerned primarily with *valid* inference. The meaning of this should be fairly clear. "Valid" means the same as "correct," or "accurate," or "sound"; and at some time or other everybody distinguishes between (usually his own) "correct" inferences and (his opponent's) "incorrect" inferences. Still, there is at least one point which calls for careful consideration. A *valid* inference is not the same thing as a *true* inference. Careful thinkers usually endeavour to make their inferences both valid and true; but it is possible for inferences to be valid without being true, or to be true without being valid. An inference is *valid* when it is reasonably justified by the evidence adduced in support of it (that is, by the judgments from which it is derived); it is *true* if it is in accord with the relevant facts, that is, if it describes the facts concerned approximately as they are. Now gamblers and others sometimes make rash inferences which turn out to be true, though they were not really valid. On the other hand, judgments are sometimes disproved by a process known as that of reduction to absurdity (*reductio ad absurdum*), that is, by drawing from them valid inferences which are absurd, that is, inferences which are valid but obviously not true. Now Logic is only concerned with the study of *valid* inference. This does not mean that Logic disregards truth. Far from it. Logic is certainly concerned with the formulation of the *true* conditions of valid inference. It is sheer necessity that compels Logic, as it compels all the sciences, to confine itself to a limited set of problems. The study of the conditions

of *valid* inference involves the study of the relations between inferences and premises (that is, the judgments from which the inferences are derived), and that is a sufficiently important task by itself. The study of the conditions of *true* inference would involve in addition the study of the truth of all possible premises—an obviously impossible task, and utterly opposed to the division of labour to which science, as well as industry, owes its advancement. The careful thinker will naturally see to it that his data, or premises, are true before he draws any conclusions from them. But for the study of inference as such it is necessary to isolate the problem of validity—to study the main types of inference, and the relations between the inferences and the premises when the inferences may be said to be justified by the premises. This involves abstraction from the truth of the premises. But then every science abstracts from something in order to simplify its problems sufficiently to make them manageable. To abstract from anything, however, is not the same thing as to reject or to ignore it utterly; it is simply not to deal with it at the same time as certain other problems are being dealt with, with the clear understanding that the neglected problems or aspects must receive adequate attention as soon as they become relevant in any actual, complex situation.

Generality. Logic is concerned with the *general* conditions of valid inference. Every actual argument relates to some *particular* problem, mathematical or astronomical, physical or chemical, political or economic, legal or moral, etc. The full consideration of an argument must consequently take into account its special subject-matter, and requires a knowledge of it If Logic were to attempt to deal with arguments

in all their detail, then (as was already suggested in another connection) it would have to absorb all the sciences, etc.—an obviously impossible and absurd enterprise. Logic, accordingly, abstracts from the special subject-matter of each argument, and confines itself to the study of the main *types*, or *kinds*, of argument, and the *general* conditions of validity concerning each type. This kind of procedure is not peculiar to Logic. Every science is like it in this respect. Like Nature herself, every science is careful of the type, not of the individual, except as a specimen of the type. At the same time Logic, like Mathematics, is more general or abstract than most sciences, and this characteristic generality of Logic, as of Mathematics, finds expression in the extensive use of symbols. Symbols in Logic stand mostly for the subject-matter to which the argument relates, but they are a device for abstracting from the peculiarities of the various kinds of subject-matter, and so help one to concentrate on the *general* character of the various types of argument, and the *general* conditions of their validity.

§ 2. *The Formalism of Logic.*

The fact that Logic is interested in the *validity* of inference, and only in the *general* conditions of such validity, may also be expressed by saying that Logic is interested in the *general relationships* between inferences and the premises from which they are derived, or by which they are justified. Such general relationships, in which the peculiarities of their terms (or subject-matter) are abstracted from, may be described as "forms," or "forms of argument"—the actual terms (or subject-matter) constituting the "matter" of the argument. Logic may therefore be said to be

concerned with "forms" of argument. Hence the occasional reference to the "formalism" of Logic. It means, however, no more than that Logic is concerned with *types* of argument, and with the *general* conditions of their validity. It means no more than that Logic is *general* and rather *abstract* in its methods and aims. In this respect, however, the difference between Logic and the Sciences is at most one of degree, not one of kind.

§ 3. *Inference and Knowledge.*

Inference is a most important pathway to knowledge. There are other avenues to knowledge, namely, perception and intuition, which give us immediate judgments as distinguished from inferred judgments or inferences. Still, the amount of knowledge that can be gleaned from personal observation and intuition is very limited, not only in quantity but also in quality. The bulk of human knowledge, and especially what is called scientific knowledge, as well as history and philosophy, is obtained by inference. So much so is this the case that to describe the different types of inference is to describe the principal ways of knowledge, and conversely to describe the principal avenues to knowledge is to describe the main types of inference. Something must therefore be said about knowledge in general and about science more particularly.

§ 4. *Knowledge and Life.*

"In the beginning was the deed." Life needs action for its maintenance. One must do things in order to live. In the lowest forms of life the actions are blind and immediate, and their success is not great. The mortality among the lowest types of animals is enor-

mous. But, as we ascend in the scale of animal existence, it becomes more and more possible to avoid many risks by the help of far-sight and fore-sight. Of such far-sight and fore-sight scientific knowledge is the highest known development. "Knowledge is power."

The humblest kinds of animals have no specialized sense-organs, and when they seek satisfaction of their needs they do so with their whole body, which is thus exposed to the risk of injury or destruction. Somewhat more developed organisms, which possess tentacles, are already at an advantage. They can examine their immediate surroundings with their tentacles alone, without risking their whole body. The possession of a special sense-organ of smell gives a still greater advantage. The animal with a sense of smell has a wider range than is possible with tentacles alone, and need not risk even its tentacles. Special sense-organs of sound and of sight render possible a more real far-sight and fore-sight. Animals so equipped cease to be entirely dependent on what is within immediate reach; they can fetch what they want from afar. They can also realize more distant sources of danger, and seek protection in time. Now human thought and human knowledge extend enormously the range of far-sight and fore-sight. Human beings can satisfy their needs by having recourse to things that are very distant, and they can prepare to meet contingencies which are very remote. Scientific knowledge represents the highest achievement in these respects. Properly utilized it should be the most potent means of protecting the human organism from danger, and supplying all that is necessary for its healthy survival.

Even the lower animals, however, play as well as toil. Their movements and other activities are not always

directed to the mere satisfaction of pressing physical needs. Activities are sometimes carried out from the sheer pleasure of action. Such play may be useful in keeping them fit, and in making them expert in the execution of necessary activities. Similarly, human beings sometimes take up sport and athletics with the deliberate aim of keeping fit and agile. For the most part, however, we take pleasure in such activities for their own sake, and without regard to ulterior practical considerations. So it is with human knowledge. When the conditions of life improve so that it ceases to be necessary to devote our whole thought and energy to the practical needs of existence, then knowledge comes to be pursued for its own sake, without regard to utilitarian considerations. In that way pure disinterested science arises.

Even science in its beginnings was intimately connected with practical needs. Geometry, for example, grew out of the practical needs of the surveyor, biology and chemistry grew out of the practical needs of the medicine man. Even now science subserves practical interests, not only in the sense that, sooner or later, practical applications are found for the purest theories of science, but also in the higher sense that scientific knowledge helps man to make a proper orientation, to take his right place in the world, and to feel at home in it. In this way science, on the one hand and philosophic reflection and æsthetic contemplation, on the other hand, render possible a more complete and more satisfactory orientation than would otherwise be attainable.

The more speculative flights of philosophic and theological reflection also aim at the satisfaction of certain human needs. But these speculations are apt to be

remote from observable reality, as is apparent from the enormous variety of such speculative ideals. It is one of the great services of science that it helps to direct and control such speculative adventures by keeping as close as possible to actual experience. Hence the agreement which one finds among men of science at all times in comparison with what is prevalent among philosophers and theologians. This is due to the fact that science seeks knowledge along certain well-defined lines. The problems attacked by philosophy are not amenable to solution along the same lines. That is why philosophic solutions are more dubious. But the results of strictly scientific inquiry set certain limits to philosophic speculation, and so help to keep philosophical hypotheses within the realm of probability.

Another great service, perhaps the highest service, which science renders, consists in the cultivation of a certain mental attitude, and in the teaching of certain methods. Both these have proved of inestimable value in the work of scientific research, and may prove equally fruitful even in such problems of life and conduct as do not strictly come within the domain of science. Huxley embraced it as one of the principal aims of his life's endeavour "to forward the application of scientific methods of investigation to all the problems of life," in the conviction that there is no other way of alleviating the sufferings of mankind.

§ 5. *Science: its Aims and its Characteristics.*

Whatever use scientific discoveries may be put to, science as such is a species of theoretical knowledge, as opposed to all forms of active skill or practical wisdom. Science as such is not an art, or a craft. It is true that

scientific experimentation often calls for a considerable degree of technical skill in the construction of suitable apparatus. The discoverer of scientific truth is often also the inventor of scientific instruments, or scientific apparatus; but the technical inventions which pave the way for scientific discovery, and the technical inventions for which scientific discovery lays the foundations, can always be distinguished from science proper, or pure science. Pure science consists essentially of theoretical knowledge.

Not all theoretical knowledge, however, is science. Science is a definite species of theoretical knowledge. There are other branches of knowledge from which science must be distinguished. The expression "science" is mostly used as a collective name for the several sciences—physics, chemistry, botany, etc. These sciences have certain characteristics in common which differentiate them from other departments of knowledge. The common characteristics of the sciences properly so called may be enumerated as follows:—

(*a*) Critical discrimination;
(*b*) Generality and system;
(*c*) Empirical verification.

A brief explanation of these common characteristics of the sciences may suffice for the present purpose.

(*a*) *Critical Discrimination.* The first requisite of all sound knowledge is the determination and the ability to get at the naked facts, and not to be influenced by mere appearances, or by prevalent notions, or by one's own wishes. Such a mental attitude is commonly described as a scientific frame of mind. It is a *sine qua non* of all science. He who is credulous enough to take things at their face value, or is so lacking in

independence and initiative that he cannot break away from customary ideas, or is so partial as to be influenced by his desires and wishes, has not the making of a man of science. Perhaps the best account of a scientific mind is to be found in Francis Bacon's flattering description of himself in his *Proemium*: "A mind nimble and versatile enough to catch the resemblances of things (which is the chief point), and, at the same time, steady enough to fix and distinguish their subtle differences; . . . endowed by nature with the desire to seek, the patience to doubt, fondness to meditate, slowness to assert, readiness to reconsider, carefulness to dispose and set in order; and . . . neither affecting what is new nor admiring what is old, and hating every kind of imposture." [1] Critical discrimination is indispensable in science, but it is really the requisite of all sound knowledge, and is not the monopoly of the man of science. The philosopher and the historian exercise it as well as the scientist. The scientific frame of mind on the part of an investigator is not by itself sufficient to make the results of the investigation a science.

(*b*) *Generality and System*. Science is not interested in individual objects, or in individual groups of objects as such. It is primarily concerned with types, kinds or classes of objects and events, of which the individual object or event is treated merely as a specimen or an instance. The aim of science is to trace order in Nature. To this end, science seeks to ascertain the common characteristics of types of objects, and the general laws or conditions of events. Each law discovered is, so to say, a thread in the essential nature

[1] *De Interpretatione Naturae Proemium*, vol. iii. pp. 518 f., in Ellis and Spedding's edition of Bacon's *Works*.

of the class of objects, or events, concerned; and the discovery of many such laws leads to a conception of the whole pattern or system. In these respects history, that is social and political history, is not a science. It is just as interesting and legitimate a study as science is, and calls for the same kind of constructive imagination and critical insight, but it is different from science. Even the history of science, although it requires considerable scientific knowledge, is a history and not a science. History is concerned with *particular* nations, or institutions, discoveries, or inventions, not with laws relating to nations and institutions, etc., generally. Such general laws would belong to ethnology, or anthropology, or sociology, or psychology, which are sciences, not to history.[1] Astronomy and geology may, at first, appear to be concerned with particular objects; and, to some extent, they may be regarded as marking a transition stage from sciences concerned only with what is general to studies concerned only with particulars. Strictly speaking, however, even astronomy and geology are largely or mainly concerned with what is general. Each stellar orbit is really a law of the sequence of positions of a planet or comet, etc.; and astronomy is also concerned with the formulation of the cycle of stages through which all stellar systems pass. Similarly, geology is concerned with the general relationships between various kinds of strata, and seeks to formulate the sequence of stages through which all continents pass.

[1] Natural History, of course, is not a history in the present sense of the term. The name "natural history" is a survival from the time when the name "history" was still used for the descriptive account of anything. Aristotle's treatise on Zoology was called *The History of Animals*; and Bacon called all sciences "histories."

(c) *Empirical Verification.* Science begins with facts of actual observation, and constantly returns to observations, in order, directly or indirectly, to check all its tentative explanations, or hypotheses. A suggested explanation which cannot, directly or indirectly, be put to the test of observation, so as to be either confirmed or confuted by it, is of no use in science. In this respect science is different from philosophy. In philosophy it is permissible and usual to put forward hypotheses which cannot be put to the crucial test of observation. True, even philosophical hypotheses are based on experience, and are intended to explain experience; but that is a different thing from being capable of confirmation, or confutation, by observation or experiment under specified conditions. The scientific hypothesis must not only account for all the observations already made of the phenomenon concerned, but must be capable of being definitely confirmed or confuted by further observations, or experiments, under specified conditions.

§ 6. *Wider Use of the Term "Science."*

In the foregoing account the term "science" is used in the stricter sense in which it is commonly employed among English-speaking men of science. The term "science" is, however, also employed in a much wider sense not only among English-speaking laymen, historians, and philosophers, but on the European Continent generally. The French term *science*, the German *Wissenschaft*, and the Italian *scienza* are commonly applied to history and philosophy as well as to those departments of inquiry to which the term "science" was restricted in the foregoing section. This wider use of the term has some advantages and removes a source

of irritation among those who are perhaps a little too anxious lest their particular researches should be deemed to be inferior to science. But the general adoption of this wider use of the term "science" would in no way remove the need of distinguishing between history, philosophy, and the sciences in the narrower sense of the term. On the European Continent history and philosophy are described as *Geisteswissenschaften*, physics, chemistry, etc., as *Naturwissenschaften*, and so on. It would be no easy task to coin, and to obtain currency for, equivalent English terms. On the whole there is a good deal to be said in favour of the stricter use of the term "science" outlined in the preceding section.

§ 7. *The Main Kinds of Reasoning.*

The reasoning by which most of our knowledge is obtained is of two main types as regards the data or the starting point from which it sets out. It may start from some assertions or statements, behind which it does not go, and seek to unfold their implications. Or it may start from observed facts and try to discover their character and explanation. The former kind of reasoning may be described as *formal*, the latter as *inductive*. In a complex argument the two kinds of reasoning may both be used—they frequently are used in conjunction. But there are occasions when the reasoning is mainly from certain given assertions or statements, not from any observed facts on which the statements may have been based, or from which they may have been derived. There are spheres of thought where the question of drawing conclusions from observed facts scarcely arises, and where the reasoning is mainly concerned with the implications of given

assertions or statements behind which it need not go. The domain of the lawyer furnishes a large class of such cases. The numerous state laws and local regulations are for the time being so many general statements which are authoritative. They have not to be discovered from observed facts in the way in which a physicist or a chemist has to discover a physical or chemical law. They are accepted as final for the time being and are applied to relevant cases by inferring their implications. The main work of lawyers and of law-courts is concerned with such problems of implication, with the determination of what exactly certain laws, regulations, or contracts really imply or do not imply. Formal inference from given propositions is thus carried on by itself to a great extent, and can therefore be studied by itself, before dealing with the additional problems which arise in connection with inductive inference, that is, inference from observed facts. It is accordingly convenient, though not necessary, to begin with the study of formal reasoning, and then proceed to the study of inductive reasoning. The former study is usually known as *Formal Logic*; the latter as *Inductive Logic*, or *Methodology*, or *the Study of Scientific Method.* Inductive reasoning frequently includes some formal reasoning. What inductive reasoning mostly aims at is the discovery of some general truth from which certain observed facts might have been inferred. Many of the propositions from which formal reasoning sets out have actually been discovered by inductive reasoning from observed facts. But, as already explained, many such premises are arbitrary (though not capricious) laws or regulations, or they may be provisional assumptions, etc., requiring no previous induction.

Just as Law constitutes a large domain (though not the only one, of course) of formal reasoning, so Science constitutes a large field for inductive reasoning (though it is by no means the only field). The so-called methods of science are in their essence either methods of inductive reasoning or auxiliaries to such reasoning. This may not seem obvious, and may require a brief explanation.

§ 8. *Scientific Methods, Technical and Logical.*

In a wide sense, any mode of investigation by which the sciences have been built up and are being developed is entitled to be called a scientific method. Broadly speaking, these methods are of two distinct kinds. On the one hand, there are the *technical* or *technological methods* of manipulating and measuring the phenomena under investigation, and the conditions under which they can be observed fruitfully. Probably it is these technical methods of manipulation and measurement that are most readily recalled by the expression "scientific methods." These technical methods are mostly different in the different sciences, and few men of science ever master the technical methods of more than one science or one group of connected sciences. On the other hand, there are the *logical* methods, that is to say, methods of reasoning according to the nature of the data obtained. These logical methods are intimately connected with the technical methods. In a very real sense the technical methods, although they are extremely important or even indispensable in many scientific investigations, are mainly auxiliary to the logical methods o· science. What is meant is this. In pure science the technical methods of science are not usually an end in themselves. They are aids either to

observation or to inference. Sometimes they render possible the observation and measurement of certain phenomena which either could not be observed and measured at all otherwise, or could not be observed so well and measured so accurately. At other times the technical methods enable the investigator so to determine the conditions and circumstances of the occurrence of the phenomena which he is investigating that he can reason about them in a definite and reliable manner, instead of merely speculating about them vaguely. (For an illustration, see, e.g., Chapter XIX, § 3.) The conjectural, highly speculative character of early science was probably due, in large measure, to the lack of suitable technical methods and scientific apparatus. However, whereas the technical methods are, for the most part, different from one science to another, the logical methods are more or less common to all the sciences. They are, moreover, the only scientific methods that can be studied with advantage by those who are not men of science, in the strict sense of the term, as well as by those who are.

§ 9. *The Function of Logic.*

The main function of Logic is to make intelligible, or to explain, the general nature of valid inference; not to enable one to argue or to reason more correctly, though it may do that also incidentally. The main purpose of every science is to enable people to *understand* things, not to *do*, or to make, them. Astronomy does not profess to construct stars or stellar systems, or to teach the stars their courses; its aim is to describe and to explain the stars and their movements. People do not wait for physiology to teach them to eat and drink, to walk and run, etc. If people could not do

these things without the aid of physiology, physiology itself could never come into existence. Similarly, people can and do reason correctly without the aid of Logic; if they could not do so, Logic itself would not exist. God (as Locke remarked) has not been so sparing to men as to make them barely two-legged creatures, leaving it to Logic to make them rational. Rather it is the native rationality of man that has made Logic itself possible. The main aim of Logic is not to teach people to reason correctly, but to explain what happens when they do reason correctly, and why some reasoning is not correct. At the same time just as a knowledge of the sciences is generally useful in some way or other, although such utility is not their main aim, so a knowledge of Logic may be, and should be, useful in checking one's conclusions, if one is not so desperately self-complacent as to be beyond all help and all improvement.

§ 10. *The Main Divisions of Logic.*

In the light of the foregoing explanations it should be clear now that the principal divisions of Logic are those usually known as *Formal Logic* and *Inductive Logic*. Under Formal Logic we shall deal with the chief problems involved in the study of the various kinds of inferences which may be drawn legitimately from given propositions. This will include an account of the general nature of propositions, of immediate inference, or the implications of isolated propositions, of mediate inference, or the implication of certain combinations of propositions, and of deductive inference, or the application of general propositions to suitable cases or classes of cases. Under Inductive Logic we shall deal with the chief problems involved

in the study of the various kinds of inference which may be drawn from observed facts in order to explain them. This will include the study of the methods of science, including the so-called inductive methods, also an account of reasoning from analogy, from circumstantial evidence, and probable reasoning, which is largely inductive in its most familiar forms, though partly also deductive in character. Lastly, various problems relating to the general character of reasoning, formal and inductive, will be discussed briefly in their appropriate places. These problems include such topics as the ultimate assumptions of all reasoning, the so-called Laws of Thought, the objective order of natural phenomena, and such auxiliaries to clear thinking as Definition. The commoner and more serious types of fallacy, that is, violation of the conditions of valid reasoning, will be referred to in the course of the exposition of the kind of reasoning in which it is apt to occur.

FORMAL LOGIC

CHAPTER II

JUDGMENT AND TERMS

§ 1. *Judgment and Proposition.*

A judgment or belief when expressed in language is commonly called a proposition. Even our private thoughts, that is, even the judgments which we do not at the time communicate to others, or put on record, are carried on largely through the medium of inarticulate language. But it is obvious that judgments cannot be explained to others, or discussed with them, except through the medium of propositions. For this reason we shall be concerned mainly with propositions, and the relations which must exist between propositions in order that one proposition (called the inferred proposition, or simply the inference, or the conclusion) may be said to be legitimately derived from another or others (called the premises, or the evidence, or the data). It should be noted, however, that the term *proposition* is used in a somewhat more extended sense than has been indicated so far. It denotes not only the verbal expression of an *actual* judgment or belief, but also the verbal expression of a suggestion, or supposition, or a merely *potential* judgment. Among thoughtful, critical people suggestions and suppositions *as such* play an important rôle. Only the uncritical and conceited dogmatist regards whatever he takes into his head as an indisputable intuition, if not as a divine inspiration. The critical person, who alone has the making of a man of science, turns such thoughts round and round, treats them as mere suggestions, or suppositions, or "propositions," and scrutinizes them with caution in the

light of the available evidence. All such suggestions, when expressed in language, are also called propositions. It is, moreover, a familiar fact that what one person firmly believes, another may as firmly disbelieve, while yet a third may regard it as a suggestion worth considering. It will be most convenient, accordingly, to use the expression *proposition* for the expressed content of any thought such as may be believed, or disbelieved, or merely understood and considered. In other words our use of the term proposition makes abstraction from the element or moment of belief, and any statement which can be true or false is called a proposition, no matter whether it is believed, disbelieved, or merely under consideration.

§ 2. *Implication and Inference.*

It should be fairly obvious that one proposition can be inferred legitimately from another proposition, or from other propositions, only when the other proposition or propositions *imply* it. In fact *implication* and *inferability* are correlative terms—to say that certain conclusions are *inferable* from certain premises, is equivalent to saying that those premises (either separately or jointly) *imply* those conclusions. The problem of inferability is therefore the same as that of implication. And the question of the general conditions of *valid inference* may be answered by considering the *implications* of the different types of propositions regarded as potential premises. It is clear from ordinary usage that when we distinguish between the *meaning* of a statement and its *implication* we really distinguish between its more obvious and its less obvious sense. That, at all events, is the way in

which we usually distinguish between what a person *says* and what he *implies* (or insinuates, etc.). The difference between *meaning* and *implication* is only a difference of degree at most, and it is not always easy to determine at what point precisely the (direct) *meaning* of a statement ends, and its (indirect) *implication* begins. For the purpose of the logical problem the distinction is of no fundamental importance, and the term *implication* may be used in an inclusive sense, covering the more obvious, direct meaning of statements as well as their less obvious, or indirect, meanings.

§ 3. *Judgments and Terms.*

It has already been stated above (Introduction, § 4) that knowledge is a power by means of which man is helped enormously in his struggle for existence, because it gives him a more effective orientation than would otherwise be possible. This new kind of intellectual orientation consists of thought or judgments. Considered biologically, the chief feature of our intellectual orientation is an enormous improvement in the extent to which we learn from past experience, not only from our own individual experience, but also from the experience of others. Learning from experience is by no means new at the human level—even young chicks learn from experience. But at the human level the process sometimes becomes clear-sighted and articulate. Objects and situations which we have already experienced, and which we have learned how to deal with, are retained in ideas, or concepts, which are helpful in new situations of the same kind. Again, even lower animals, certainly chimpanzees,

are taught by experience to apprehend vaguely that certain things are connected with one another, or are dependent on one another—say, the falling of bananas out of a certain basket with the pulling of a string attached to the basket. At the human stage, however, such apprehension becomes clearer, and is articulated in general ideas of laws of interdependence, etc., which can be applied to real or imaginary situations with a foresight of the probable result.

In the simplest kinds of judgments what happens is this: something confronting us, and in some way of interest to us, is recognized as being an object, or quality, etc., of a certain kind with which we are already familiar from previous experience, and of which we accordingly have a concept (or idea); or some concept which first suggested itself is denied of the object in question, usually because some other concept seems to fit better. Such judgments when expressed in language may assume the form of such simple utterances as "Rain," "Fog," "A rainbow," "Cold," "Dark," "Not foggy," etc. What is expressed, in each of these cases, is the concept under which the observed object, etc., is brought, or by which it is apprehended or interpreted. The concept by means of which the object, or situation, etc., is recognized, or interpreted, is called the *predicate*; the observed object, situation, etc., which called for recognition, or interpretation, is called the *subject*. In the simplest cases, like the above, the subject is not expressed in language at all; but when it is expressed in language (by means of a pronoun, a noun, or nominal phrase) the verbal expression is also called the subject. In somewhat more developed judgments the object, or situation, etc., requiring further elucidation is not

apprehended so vaguely as to be inarticulate except for the predicate, but is at once apprehended under one concept, though still requiring further elucidation with the aid of other or more determinate concepts. In such cases the subject, as well as the predicate, is expressed in language. Subjects and predicates (also their verbal expressions) are called the *terms* of the judgment (or of the proposition). And we are so accustomed to propositions with at least two terms (subject and predicate) that judgments which would more naturally be expressed by means of the predicate only, have a dummy subject added to them. Hence such impersonal propositions as "It is foggy," or "It is cold," etc., instead of merely "foggy," "cold," etc. The *judgment* itself, of course, never involves less than two terms; only in the simplest cases, the subject is really inexpressible in language, because it is apprehended too vaguely, and so its *verbal expression* is more natural if without an expressed subject.

Judgments and propositions are of varying degrees of complexity. The more complex ones involve three, four, or even more terms. In fact, the more complex propositions are best treated as composed of simpler propositions, related in certain ways, just as the simpler propositions are composed of terms. But it is important to realize from the first that intellectual activities are essentially continuous, not discrete. A judgment is not produced by putting together two discrete terms, nor are more complex judgments and inferences produced by putting together several discrete judgments. The whole process is much more complex than that; it is more like a continuous growth in which the comparatively simpler objects pass into more and more complicated and more differentiated

wholes by the assimilation of new materials. It is important to bear all this in mind, as an erroneous view is easily encouraged by our pre-occupation with propositions, which in a sense really are put together from discrete words or letters. In our actual mental experience what happens is quite different from such mere addition or juxtaposition of separate units. There is always a continuum—consisting of a comparatively vague background (or hinterland) out of which some objects only just emerge more definitely, while others are in the focus of consciousness, and are most clear and distinct of all. These are psychological matters with which we are not directly concerned; but it is desirable to avoid misapprehensions which may easily distort one's conceptions of topics which are of more immediate logical interest.

We may proceed now to the consideration of the principal types of propositions and their implications.

CHAPTER III

CATEGORICAL PROPOSITIONS AND THEIR IMPLICATIONS

§ 1. *The General Character of Categorical Propositions.*

In the simpler, and commonest, type of proposition a predicate is simply affirmed or denied of a subject By "simply" is here meant unconditionally, or without reservation, and without implying any necessary connection between the terms of the proposition. Such propositions are called *categorical* propositions. The following may serve as examples of such propositions. *The earth is a planet. The earth is not flat. All planets move in elliptical orbits. No planets are fixed. Some stars are self-luminous. Some planets are not self-luminous.* If we abstract from the special character of the terms, and the quantity of the subject (that is whether one, some, or all are referred to) then the general character of categorical propositions may be represented by the formulæ *S is P* and *S is not P*, where *S* stands for any *subject*, and *P* for any relevant *predicate*. These and similar symbolic formulæ are usually called *forms*, because they abstract from the subject-matter, or actual terms, of propositions—that being the only way in which *types* of propositions can be studied. But it should be noted once for all that it is always assumed that actual propositions have definite terms, even if their special character is not at the moment under consideration. There is no such thing as a proposition which has form but no subject-matter, any more than there is such a thing as a proposition which has subject-matter and no form. But the forms of propositions may be studied to a con-

siderable extent apart from their actual terms, and terms may be studied to some extent apart from the propositional forms in which they occur.

§ 2. *The Quality of Categorical Propositions.*

It will be observed that in the above examples of categorical propositions the predicate is *affirmed* of the subject in some cases, and *denied* in others. The difference between *affirmation* and *negation* is called a difference of *quality*, and it is the only difference in respect of quality. Every proposition is either *affirmative* or *negative*, according as the predicate is affirmed or denied of the subject. Thus, to revert to our previous examples, *The earth is a planet, Some stars are self-luminous, All planets move in elliptical orbits*, are all *affirmative* propositions; on the other hand, *The earth is not flat, Some planets are not self-luminous, No planets are fixed*, are all *negative* propositions.

The relation between affirmative and negative propositions calls for some consideration. On the one hand, it is true, in a very real sense, that the difference between affirmation and denial is fundamental; it cannot be effaced by reducing either to the other. Given an unambiguous *subject* and an unambiguous *predicate*, say *S* and *P*, then there are three possibilities. One may affirm *P* of *S*, and the result will be an affirmative proposition, *S is P*. Or one may deny *P* of *S*, and the result will be a negative proposition *S is not P*. Lastly, one may not know whether *P* should be affirmed or denied of *S*; but in this case the result is, for the time being, only a problem, *Is S P*?—a question, not a proposition. Now of the two possible propositions, *S is P, S is not P* (*S* and *P* being unambiguous in every way), the second is really a rejection of the

former. It comes to the same thing whether one says, "I disbelieve *S is P*," or whether he says, "I believe *S is not P*." Now *belief* and *disbelief*, it should be obvious, are fundamentally different and incompatible attitudes towards the same suggestion. To that extent, *affirmation* and *negation* are fundamentally different from each other.

In some respects, however, the difference between affirmative and negative propositions is not one of fundamental importance, but rather one of convenience. Namely, what is essentially the same suggestion, or belief, may be expressed either in an affirmative proposition, or in a negative proposition, according to circumstances. But, of course, the *terms* will have to be different to some extent. For instance, the negative proposition *No planets are fixed* means the same thing as the affirmative proposition *All planets move*; similarly the affirmative proposition *Air is transparent* means the same thing as the negative proposition *Air is not opaque*; the negative proposition *No railway tickets are transferable* means the same as the affirmative proposition *All railway tickets are non-transferable*; and so on. Or, to take an example already used in another connection, the negative statement, "The proposition *S is P* is not true" means the same thing as the affirmative statement, "The proposition *S is not P* is true"—a very different thing from maintaining both that *S is P* and that *S is not P*, which (as has already been explained above) would be impossible, so long as *S* and *P* are quite unambiguous.

§ 3. *The Quantity of Categorical Propositions.*

The examples of categorical propositions already given will have shown that they vary not only in the

way in which the predicate is asserted of the subject, but also as regards the extent of the subject of which the predicate is asserted. For example, in the proposition *The earth is a planet* the subject is a *single* object; in the proposition *All planets move in elliptic orbits*, or *No planets are fixed*, the subject is a *class* or *kind*, and the assertion is made of *any* and every member of it; in the proposition *Some stars are* (or *are not*) *self-luminous* the subject is indeterminate, the assertion is made of one object at least, it may be of several, or of many objects; it may be true of the whole class or kind. These differences are called differences of *quantity*. Propositions like the first of the above propositions are said to be *singular*; those like the second and third are called *general*; those like the last are called *particular*.

There are several minor points worth noting. (1) A proposition remains *singular* even when the subject of which the assertion is made consists not of a solitary object but of a group of objects, so long as the group is treated as a group, that is, as one complex object. For example, *The British Museum Library consists of several million books and pamphlets* is a singular proposition, because, although the subject is so numerous, the items are all regarded as forming one group, or collection. Similarly with the proposition *All the major planets are eight in number*. The subject is treated as one group, and the predicate is asserted of the group as such, not of each of the planets separately. (2) As the last illustration may have suggested, it is important to distinguish carefully between a *group* and a *class* (or *kind*, or type). A *group* is a collection of several, or many, similar objects—multiplicity or *quantity* is an essential feature

of a group; a class (in its scientific meaning, not as applied to school forms or standards), or kind, is so called in virtue of certain qualities or characteristics which distinguish it from other kinds of objects, etc. There are biological classes (or kinds) which are represented by the remains of a solitary specimen each; but mere number, or quantity, is of no consideration in the case of *kinds.* An assertion made of a class, or kind, as such, is made of that combination of qualities, and therefore of each thing that has those qualities—whether there are many such things, or only a few, or only one such thing, is immaterial. But an assertion made of a group may only be meant of the group as such, not of its individual components Compare, for instance, the propositions *All planets move in elliptic orbits* and *All the major planets are eight in number.* The former is a general proposition concerning a class; the latter is a singular proposition concerning a group. (3) Singular and general propositions, though different from each other in the way indicated, have something important in common. In contrast with particular propositions they are both of them definite, or determinate. The particular proposition is inevitably indefinite, or indeterminate. For example, suppose I know that *Some stars are self-luminous,* from this alone I cannot tell whether the next star I observe, or hear of, is or is not self-luminous. That is the element of indefiniteness and uncertainty. But both general and singular propositions are free from this. Hence they are usually grouped together as *universal* propositions—what they assert they assert of the whole extent of their subject (be that subject singular or innumerable), not of some indefinite part of it.

§ 4. *The Four Kinds of Categorical Propositions.*

If now we combine the main differences of quality and quantity considered above we obtain four main kinds of categorical propositions, namely, (1) *universal affirmative*, (2) *particular affirmative*, (3) *universal negative*, and (4) *particular negative*. These four kinds of categorical proposition may be represented respectively by the following formulæ: (1) *Every S is P*, if general (*This S is P*, if singular); (2) *Some S's are P*; (3) *No S is P* (*This S is not P*, if singular); and (4) *Some S's are not P*. It is convenient in some ways to have a brief designation for each of them and so they are usually referred to, respectively, as (1) *A*, (2) *I*, (3) *E*, and (4) *O* propositions. These letters are derived from the Latin words *AffIrmo* (I affirm) and *nEgO* (I deny). And the symbolic formulæ for (1) *Every S is P*, (2) *Some S's are P*, (3) *No S is P*, (4) *Some S's are not P* are respectively (1) *SaP*, (2) *SiP*, (3) *SeP*, (4) *SoP*.

The meaning of these four propositional forms can be formulated in a perfectly unambiguous manner. There are other ways of expressing their respective meanings, and it is not necessary, or even desirable, to confine oneself pedantically to these four propositional forms. The important point to remember is that the other forms of statement are not so free from ambiguity, and, if and when any doubt may arise about their precise meaning, it is best to express one's meaning in these unambiguous propositional forms. It is possible to express unambiguously any statement in these propositional forms, without doing violence to the King's English.

§ 5. *Relations between Terms in Categorical Propositions.*

Before proceeding to determine the precise meanings of the four propositional forms, there are some minor points to be cleared up. In different propositions (whatever their form may be—whether *A*, *I*, *E*, or *O*) the terms may be related in different ways. Sometimes the relation between the subject and the predicate is that between a thing (or things) and its (or their) attributes. For example, *Civilized people are tolerant*, *Oppressed people are discontented.* In other cases the relation is that between attributes. For instance, *Hopefulness induces cheerfulness*, *Despondency spells failure.* In still other cases the relation is that between classes (or kinds) of objects. For example, *The Norwegians are Scandinavians*, *Whales are mammals*, *Triangles are rectilinear figures.* And there are other relations possible, which need not detain us. Now for some purposes the differences between these relations may be of importance—say, from the point of view of psychology, or from the point of view of epistemology. But they are of no real importance for Logic, that is, for the study of valid inference. For our purpose no violence is done if, on occasion, a proposition expressing a relation between things and attributes, or a proposition expressing a relation between attributes, is so modified as to express a relation between classes or kinds. The main difference introduced is that of less abstractness (or more concreteness) of expression. Logically there is no fundamental change involved, because classes or kinds (as already explained) are distinguished or characterized by their attributes. So, for example, there is no violence done if some of the above-mentioned propositions are restated as follows: *Civilized people are tolerant people*, *Hopeful people are*

cheerful people, Despondent people are people who fail. This device of restating propositions expressed partly or wholly in terms of attributes in propositions expressed in terms of things, or classes, or kinds of things, has the advantage of simplifying the formulation of the meaning and implication of categorical propositions.

§ 6. *The Distribution of Terms in Categorical Propositions.*

Assuming, then, that the terms of categorical propositions may be legitimately interpreted in *extension*, that is, in terms of things, or classes of things (in contrast with *intension*, that is, in terms of attributes), we must next consider the question of the *distribution of terms* in categorical propositions. A term is said to be *distributed* when reference is made explicitly to its whole extent, that is, to the whole class of objects which it denotes; otherwise the term is said to be *undistributed*. In *E* propositions both terms are distributed—*SeP* means that the whole class *S* is outside of the whole class *P*. In *A* propositions the subject is distributed, but the predicate is undistributed—*SaP* means that the whole class *S* is included in the class *P*, but while the class *S* may sometimes coincide with the whole class *P* that is not always so, and must not be assumed to be so except on additional evidence. Compare, for instance, the proposition *All equilateral triangles are equiangular triangles*, where the two classes coincide, with the proposition *All Danes are Scandinavians*, where the two terms do not coincide. In *O* propositions, the predicate is distributed, but the subject is undistributed—*SoP* means that one or more *S's* at least are outside the whole class *P*.. In *I* propositions neither *S* nor *P*

is distributed. The distribution of terms in categorical propositions may, therefore, be summed up as follows: *Only Universal Propositions distribute their subject, and only Negative Propositions distribute their predicate.*

§ 7. *General Rule of Formal Inference Concerning the Distribution of Terms.*

It is important to remember the distribution of terms in the four categorical propositional forms, if only because of a certain general rule which applies to all formal inference from given propositions (as distinguished from inductive inference from observation, etc.). The rule is this: *No term that is undistributed in the premises may be distributed in the conclusion, unless the Laws of Thought warrant it* The common sense of this rule is obvious. If a given term is not distributed in the premises, we have no evidence relating to the entire class which it denotes. But if the conclusion distributes that term, it asserts something about that whole class, and so goes beyond the evidence.

We may now proceed to consider the implications of the several forms of categorical proposition.

CHAPTER IV

IMMEDIATE INFERENCE—OPPOSITION

§ 1. *Kinds of Immediate Inference.*

By an *immediate inference* is meant whatever conclusion may be drawn from a single proposition, as distinguished from what may only be inferred from two or more propositions jointly. The problem of the determination of the various types of immediate inference falls into two parts. In the first part are considered the various inferences which may be drawn from a given proposition in terms, or in respect, of another proposition having the same subject and the same predicate as the given proposition, but differing from it in respect of quality, or of quantity. This part is known as *the doctrine of the opposition of propositions.* The second part deals with inferences which may be drawn from a given proposition involving certain other subjects and predicates than those of the given proposition. This part is known as *the doctrine of eductions.*

Both types of immediate inference, however, rest on certain fundamental assumptions which must first be considered briefly.

§ 2. *The Laws of Contradiction and Excluded Middle.*

The assumptions in questions are included among the so-called Laws of Thought, and are known as the *Law of Contradiction* and the *Law of Excluded Middle.* According to the Law of Contradiction the same predicate cannot be both affirmed and denied of precisely the same subject—*S* (the same *S*) *cannot both be P and not be P.* According to the Law of Excluded Middle,

a given predicate must either be affirmed or denied of a given subject—*S* ***must either be P or not be P***, it cannot be neither, just as it cannot be both. These are fundamental assumptions on which all consistent thinking rests; all apparent exceptions rest on misunderstandings, or on quibbles. With the aid of these Laws of Thought we may now consider the opposition of propositions.

§ 3. *The Formal Opposition of Categorical Propositions.*

As already remarked, we are concerned here with the relations between propositions having the same subject and predicate, and differing only in form, that is, in quality or in quantity. Now there are only four such propositional forms—*SaP*, *SiP*, *SeP*, *SoP*—and we have to determine the relation of each to the others. Let us consider them in turn.

First *SaP*. According to it *P* is affirmed of every *S* without exception. Therefore if *SaP* is true *SiP* must be true; if it were not, that is, if it were possible not to affirm *P* of one or more *S*'s, it would be impossible to affirm *P* of every *S*, that is, *SaP* could not be true. Again, *SaP* implies the falsity, or rejection, of *SeP*, for if *SeP* could be true at the same time as *SaP* then the same subject, each *S*, would at once ***be*** and ***not be P***; and this would be a violation of the Law of Contradiction. Therefore, *SaP* implies the falsity of *SeP*. Similarly, *SaP* implies the falsity of *SoP*. For if both could be true together, then some *S*'s would both ***be P*** (because *SaP*) and ***not be P*** (because *SoP*); and this is excluded by the Law of Contradiction. Thus *SaP* implies *SiP*, but excludes *SeP* and *SoP*.

Next *SiP*. *SaP* cannot be inferred from *SiP*, for

the inference must not distribute a term (S in this case) not distributed in the premise; but, of course, SaP, though not inferable from SiP may be true at the same time. Again, SiP excludes SeP, for if both could be true, then some S's would both *be P* (because SiP) and *not be P* (because SeP); and this would violate the Law of Contradiction. But SiP neither implies nor excludes SoP. It does not imply SoP because when SiP is true SaP may also be true, in which case SoP could not be true. It does not exclude SoP, because the "some S's" which are asserted to *be P* may be different S's from those which are asserted *not to be P*; and this would not involve a violation of the Law of Contradiction. They are therefore, simply compatible; neither inferable one from the other, nor exclusive one of the other.

Next SeP. This, as we have already seen, is inconsistent with both SaP and SiP. On the other hand, it implies SoP for the same reason that SaP implies SiP.

Lastly SoP. This, as we have already seen, is inconsistent with SaP, is compatible with SiP, and is implied by SeP.

So far we have considered the question whether these various pairs of propositions can or cannot be true at the same time. To complete our survey of their mutual relationships we must still consider whether they can or cannot be false at the same time. Or how does the falsity, or the rejection, of any one of them affect the truth or falsity (the acceptance or the rejection) of the others?

Let us begin with SaP. Suppose it is not true. This means that it is incorrect to affirm P of every S—in other words, there are at least some S's (one or

more) of which one should not say that it *is P*. But, according to the Law of Excluded Middle, anything must either *be P* or *not be P*. Consequently, of those *S's* (one or more) of which it is incorrect to assert that they *are P*, one must assert that they *are not P*, that is, *SoP*. Thus the falsity, or rejection, of *SaP* implies the truth, or the acceptance of *SoP*. On the other hand, *SiP* and *SeP* may either of them be true (not both, of course) when *SaP* is false, one simply cannot tell.

Next take *SiP*. Suppose it is false. This means that it is incorrect to say of even one *S* that it *is P*. Consequently, according to the Law of Excluded Middle, it is right to say of every *S* that it *is not P*, or *SeP*. Thus the falsity, or rejection, of *SiP* involves the truth, or acceptance, of *SeP*, and therefore also of *SoP*, which, as already explained above, is implied by *SeP*. Obviously the falsity, or rejection, of *SiP* implies *a fortiori* the falsity, or rejection, of *SaP*—if it is incorrect to say of even one *S* that it *is P*, it must be even more incorrect to say of every *S* that it *is P*.

Consider next *SeP*. Suppose it is untrue. This means that it is incorrect to assert of every *S* that it *is not P*, or, in other words, that there is at least one *S* of which it is incorrect to assert that it *is not P*. If so, then, by the Law of Excluded Middle, it is correct to assert of at least that *S* that it is *P*, or *SiP*. Thus the falsity, or rejection, of *SeP* implies the truth, or the acceptance, of *SiP*. On the other hand, it carries no implication with regard to *SoP* or *SaP*, either of which (though not both) may be true, or not if *SeP* is not true.

Finally, *SoP*. Suppose it is false. This means

that it is incorrect to assert of even one S that it ***is not*** P Therefore, by the Law of Excluded Middle, it is correc to assert of every S that it ***is*** P SaP. Thus the rejection, or falsity, of SoP implies the acceptance, or truth, of SaP, and therefore of SiP. It also implies the rejection, or falsity, of SeP—what cannot be asserted of ***any*** S can certainly not be asserted of *every* S

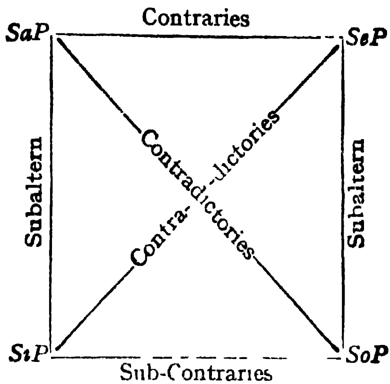
It may be helpful to sum up the foregoing results in the following table :

TABLE OF RELATIONS OR OPPOSITIONS.

Given	SaP	SiP	SeP	SoP
SaP true	—	true	false	false
SiP true	not known	—	false	not known
SeP true	false	false	—	true
SoP true	false	not known	not known	—
SaP false	—	not known	not known	true
SiP false	false	—	true	true
SeP false	not known	true	—	not known
SoP false	true	true	false	—

Some of the above relations between propositions having the same subject and predicate have received special names. They are summarized in the following diagram, which is known as

The Square of Opposition.



The following points should be specially noted:

Contraries. *SaP* and *SeP* are extreme opposites, and do not between them exhaust all possibilities. They cannot both be true; but they may both be false, namely, when both *SiP* and *SoP* are true. Singular propositions and particular propositions have no formal contraries, only contradictories.

Sub contraries. The relationship between sub-contraries must be carefully distinguished from that between contraries, as they are the precise reverse of each other. Of *SiP* and *SoP*, one must be true, and both may be true—or both cannot be false, and neither need be.

Contradictories. Of the two propositions *SaP* and *SoP* both cannot be true, but one of them must be true. In other words, they are mutually exclusive (without being extreme opposites) and collectively exhaustive. The same holds good of *SeP* and *SiP*.

CHAPTER V

IMMEDIATE INFERENCE- EDUCTIONS

§ 1. *Eductions.*

In the doctrine of opposition we were concerned with the relations between propositions having precisely the same subject and predicate, but differing in quality or in quantity. We now proceed to consider the implications of the four types of categorical proposition beyond these limits. A proposition having S for its subject and P for its predicate may imply a proposition having P for its subject and S for its predicate, or propositions containing the contradictories of S and P. Such propositions are called *eductions.* There are two principal types of eduction, and five derivative forms. The principal types are known as the ***converse*** and the ***obverse***; the other forms are obtained (when they can be obtained) by combining, or repeating alternately, the steps by which the converse and the obverse are obtained. The student who has no special distaste for dealing with a few easy symbols will find no difficulty with the Eductions. Above all he should endeavour to understand the steps by which the results are obtained, and not put his trust in merely remembering the results. Memory is a poor substitute for intelligence.

§ 2. *Contradictory Terms and their Symbols.*

It will be convenient at this point to say something about pairs of contradictory terms and their symbols, as the proper understanding of these simple matters is necessary to the real grasp of what follows.

Our thoughts and discussions usually have reference to some more or less well-defined class of objects, or qualities, etc., say, the inhabitants of the British Empire, economic goods, portraits, or colours, etc. Now all objects are what they are primarily because of the characteristics which they have, and the relations in which they stand; and for that reason all objects can be designated by means of positive terms, that is terms which imply or suggest what the objects are or have. But all finite objects, just because they are limited, are also without innumerable characteristics, for (as Spinoza says) *omnis determinatio est negatio* ("every limitation is a negation"); and for that reason they can also be referred to by means of negative terms, that is, terms which imply or suggest the absence of certain characteristics No doubt it would be possible to refer to anything by means of positive terms. But there are such things as human needs and purposes, in relation to which some things, etc., are suitable while others are not. In this way many objects come to be designated by means of *negative terms*, that is, terms whose primary function it is to draw attention to what an object *is not*, or *has not* For example, it may be necessary to estimate the war-strength of the British Empire, and for that purpose its inhabitants may be distinguished as *British subjects* (positive term) and *aliens* (negative term). Similarly, goods may be distinguished as *British* (positive term) and *foreign* (negative). It is obvious that the negative terms designate objects which have positive characteristics, just as the positive terms refer to objects which also lack all sorts of characteristics; and that positive and negative designations change places for different purposes. In France, for instance,

British subjects are aliens, and British goods are foreign. Similarly, if one particularly wants a red tie, all other ties are simply "not red," whereas if one particularly wants a blue tie, blue ties cease to be regarded merely as "not red," and the red tie becomes merely "not blue." Now any pair of terms, by means of which a class of objects is divided into two mutually exclusive and collectively exhaustive classes, are called *contradictory terms*. Of a pair of contradictory terms one is *positive*, namely, the one which indicates the presence of the characteristic (or group of characteristics) in which one is interested at the moment, while the other is *negative*, namely, the one which indicates the absence of that characteristic. Sometimes the negative term is verbally expressed by prefixing "non" to the positive term—*non-productive, non-transferable, non-juror, non-payment, non-conformist, non-commissioned officer, non-conductor*, etc. At other times the negative terms in use are not so constructed —for instance, *alien, foreign, rigid, opaque*, etc. But it is always permissible and possible to use a negative term of the former type in place of one of the latter type—for example, *non-British* (or *non-French*, etc.) for *alien*, or *foreign*; *non-flexible* for *rigid*; *non-transparent* for *opaque*, and so on

Now a pair of contradictory terms when represented by symbols will naturally be represented in such a way that the symbols can at once be recognized as contradictory. The simplest way to this end would appear to be to represent the positive term by a positive symbol, say S, P, etc., and the negative term by a negative symbol, *non-S*, *non-P*, etc., or, more conveniently, $\bar{S}$ (= *non-S*), $\bar{P}$ (= *non-P*), etc. This is always possible. Sometimes it is also convenient;

but not always. It is usually more convenient to represent the terms of a given proposition by means of positive symbols, even when those terms are negative. It is more convenient because it is easier in that case to see at a glance the relation of the given proposition to other propositions (immediate inferences, etc.). And there is no objection to doing so. All one has to remember is that just as in Algebra x, y, z, etc., may represent negative quantities, so in Logic S, P, etc., may represent negative terms. The main thing is that S and $\bar{S}$, P and $\bar{P}$, etc., are pairs of contradictory terms. But S and $\bar{S}$ are just as contradictory when S represents the negative term and $\bar{S}$ the positive term as when *vice versa*.

§ 3. *Alternative Formulation of the Laws of Contradiction and of Excluded Middle.*

In the light of what has just been said about contradictory terms, it is also possible now to give alternative formulations to the Laws of Contradiction and Excluded Middle. It will be convenient to give these alternative formulations at this stage because they will be helpful in connection with the doctrine of Eductions which will be discussed next.

The formula for the Law of Contradiction, as given in the preceding chapter, is S *cannot both be P and not be P*, and the formula for the Law of Excluded Middle is *S must either be P or not be P* As already explained in an earlier part of the book, what happens in a categorical proposition may be described thus: some subject is related in some way to a class. Let S be the subject and Q the class. Then the class Q can usually be conceived as consisting of two mutually

exclusive, and collectively exhaustive, sub-classes, say P and $\bar{P}$, according to the purpose of the thinker. The sub-classes P and $\bar{P}$ being related as they are, if Q is relevant at all in the consideration of S, then S is either in sub-class P or in sub-class $\bar{P}$, and cannot be in both. We may accordingly also express the *Law of Contradiction* in the formula, *S cannot be both P and* $\bar{P}$; and the *Law of Excluded Middle* may be expressed in the formula *S must be either P or* $\bar{P}$.

Armed with these formulæ we may now proceed to consider the different kinds of eductions And first of all the fundamental types of eduction which are known as Obversion and Conversion.

§ 4 *Obversion.*

The obverse of a given proposition is a proposition implied by it, having the same subject but a contradictory predicate. Symbolically, if the terms of the given proposition are $S-P$, those of its obverse will be $S-\bar{P}$. Every type of categorical proposition has an obverse of opposite quality. Thus SaP implies $Se\bar{P}$, for, by the Law of Contradiction, if every S is P, no S can be $\bar{P}$. Similarly, SiP implies $So\bar{P}$, for, by the same law, the one or more S's which are P cannot be $\bar{P}$. Again, SeP implies $Sa\bar{P}$, for, by the Law of Exclud d Middle, S must be either P or $\bar{P}$, therefore if no S is P every S must be $\bar{P}$. Similarly, SoP implies $Si\bar{P}$, for, by the same law, the one or more S's which are not P must be $\bar{P}$.

It should be noted that a proposition and its obverse are equivalent, or reciprocal, that is, each

implies the other. We have already seen, and we shall see again in due course, that a proposition may imply another proposition which does not imply it in return. For example, SaP implies SiP, but SiP does not imply SaP. Similarly, SeP implies SoP, but SoP does not imply SeP. The sign " = " should be reserved for equivalent propositions, and a single arrow, pointing in the right direction, for one-sided implication. Thus $SaP = Se\bar{P}$, $SiP = So\bar{P}$, $SeP = Sa\bar{P}$, $SoP = Si\bar{P}$; but $SaP \longrightarrow SiP$, $SeP \longrightarrow SoP$.

Another point to be noted is that if the obverse of a given proposition is itself obverted we simply get back to the original proposition. So that no new result can be obtained by merely repeating the process of obversion. This should be remembered when dealing with the more complex eductions.

§ 5. *Conversion.*

The converse of a given proposition is a proposition implied by it, but having its subject for predicate, and its predicate for subject. In other words, the terms of the given proposition and those of its converse (if it has a converse) are related as $S—P$ to $P—S$. Not every proposition, as we shall see soon, has a converse.

It has already been explained in an earlier part of the book, that the subject of a proposition is the comparative starting-point in thought, it is what is felt to need explanation, or elucidation. Now what is the starting-point in thought on one occasion, or to one person, may not always be so. S may be the starting-point on one occasion, and P its elucidation;

but P may be the starting-point on another occasion, and S its elucidation. In geometry, for example, we at one stage try to find out whether "equilateral triangles are equiangular" ($S—P$), at another stage we want to ascertain whether "equiangular triangles are equilateral" ($P—S$), and so on. The question we have to consider here is simply this. Given a proposition in which something is asserted about S in terms of P, what does it imply about P in terms of S, by way of immediate inference, that is, in the absence of any other evidence?

Let us begin with SeP. This may be said to mean that the whole class S is outside of, or different from, the whole class P. If we express precisely the same relation from the point of view of P, instead of from the standpoint of S, we get PeS. Thus $SeP = PeS$. Again, SiP means that some S's are included in the class P, and this means that they are identical with some P's. What we really have, then, are objects which are both S and P, and which can therefore be described indifferently as some S's which are P, or as some P's which are S. Thus $SiP = PiS$. It will thus be seen that E and I propositions can be converted simply—one may just transpose their terms without changing in any other way the character of these propositions. It is different with A and O propositions. SaP means that the whole class S is included in the class P, that is, is identical with some indefinite part of the class P. The two classes may occasionally coincide, as for instance in the statement *All equilateral triangles are equiangular triangles*, but they may not, as, for example, in the statement *All rectangles are parallelograms.* What we always have when SaP is true is a class of objects which are both S and P,

which consists of the whole class S, but may not consist of all P's, though it must include at least some P's. P, as we have seen in a preceding chapter, is undistributed here. Therefore, in the absence of additional evidence, P may not be distributed in the conclusion, that is, we must not make SaP imply PaS, but only PiS. Thus $SaP \longrightarrow PiS$. This is called conversion by limitation, that is, by limiting, or restricting, the (universal) quantity of the original proposition—in contrast with *simple* conversion, where the given proposition and the converse have the same quantity. Lastly, SoP has no converse at all. To convert SoP into PoS would be to distribute, in the inference, a term (S) undistributed in the premise, and that is not permissible. The fact is that PaS may be true at the same time as SoP. For example, *Some rectangles are not squares*, yet *All squares are rectangles*, or *Some Europeans are not Swedes*, but *All Swedes are Europeans*. Hence SoP cannot imply PoS, which is the contradictory of PaS. Thus SoP has no converse.

It should be noted that when the converse of a given proposition is itself converted then in the case of E and I propositions we simply come back to the original proposition, while in the case of A propositions even that much is not achieved, for we get SiP instead of SaP. Therefore, in the case of conversion, as in the case of obversion, no new result can be obtained by merely repeating the process. The only way of obtaining new eductions is by applying the processes of obversion and conversion alternately, each to the result of the other. That, indeed, is the way in which the other eductions are obtained.

Before proceeding farther it may be helpful to tabulate the eductions dealt with so far.

§ 6. *Table of Principal Eductions.*

TABLE OF PRINCIPAL EDUCTIONS.

Original $S—P$	Obverse $S—\bar{P}$	Converse $P—S$
SaP	$Se\bar{P}$	PiS
SiP	$So\bar{P}$	PiS
SeP	$Sa\bar{P}$	PeS
SoP	$Si\bar{P}$	None

Note.—No significance should be attached to the fact that the symbols S, P, $\bar{S}$, $\bar{P}$, etc., are sometimes straight and sometimes slanting (or in italics). That is a mere accident.

CHAPTER VI

IMMEDIATE INFERENCE—DERIVATIVE EDUCTIONS

§ 1. *Conceivable Eductions.*

In the preceding chapter it was shown that all propositions imply an obverse the terms of which are related to the terms of the original as $S-\bar{P}$ to $S-P$, and that some propositions have a converse, the terms of which are related to those of the original as $P-S$ to $S-P$. Let us now consider in a purely abstract manner what other implications are conceivable involving the terms of a given proposition and their contradictories. The terms in question will be four in number, namely S, P, $\bar{S}$, $\bar{P}$, of which S and P represent the subject and predicate of the given proposition. Let us omit merely tautological statements, such as S *is* S, or S *is not* $\bar{S}$, and self-contradictory statements like S *is not* S, or S *is* $\bar{S}$. We are then left with the following conceivable combinations of terms for conceivable additional eductions, namely, $P-\bar{S}$, $\bar{P}-S$, $\bar{P}-\bar{S}$, $\bar{S}-P$, $\bar{S}-\bar{P}$. If we add to these the original $S-P$, the obverse $S-\bar{P}$, and the converse $P-S$, we obtain the following table of conceivable combinations of terms of propositions in relation to any given proposition. As there can be no harm in christening these various conceivable combinations before studying them more closely, names are added in all cases.

1. $S - P$ original proposition.
2. $S - \bar{P}$ obverse.
3. $P - S$ converse.
4. $P - \bar{S}$ obverted converse.
5. $\bar{P} - S$ contrapositive.
6. $\bar{P} - \bar{S}$ obverted contrapositive.
7. $\bar{S} - P$ inverse.
8. $\bar{S} - \bar{P}$ obverted inverse.

Of these combinations it will be seen that Nos. 4, 6. and 8 are each related to its immediately preceding combination (Nos. 3, 5, and 7 respectively) in exactly the same way as the obverse (No. 2) is related to the original proposition (No. 1). They are accordingly called each the obverse of the preceding form (obverted converse, etc.). If the converse can be obtained, then there is no difficulty in getting its obverted form, since every proposition, as we have seen, has an obverse. Similarly, if and when the contrapositive and inverse forms (Nos. 5 and 7) can be obtained, there will be no difficulty in determining their obverted forms (Nos. 6 and 8).

§ 2. *Actual Derivative Eductions.*

We have now to ascertain which of these conceivable eductions are implied by, or can be derived from, the four types of categorical propositions. The obverse and the converse have already been dealt with, and the results are tabulated at the end of the foregoing chapter. Keeping an eye on that table, it will be seen that by obverting the converse we obtain the following results: $SaP \rightarrow PiS = Po\bar{S}$; $SiP = PiS = Po\bar{S}$; and $SeP = PeS = Pa\bar{S}$. Similarly by converting the obverse we obtain the following contrapositives

(No. 5): $SaP = Se\bar{P} = \bar{P}eS$; SiP has none, because its obverse ($So\bar{P}$) being an O proposition cannot be converted; $SeP = Sa\bar{P} \longrightarrow \bar{P}iS$; $SoP = Si\bar{P} = \bar{P}iS$. By obverting the contrapositives we get the obverted contrapositives (No. 6) as follows: $SaP = \bar{P}a\bar{S}$; $SeP \longrightarrow \bar{P}o\bar{S}$; $SoP = \bar{P}o\bar{S}$. It only remains now to determine the inverse forms (Nos. 7 and 8). These forms can only be obtained by converting either the obverted converse (No. 4) or the obverted contrapositive (No. 6). Now the obverted converse of SaP and SiP being O propositions cannot be converted; that of SeP is $Pa\bar{S}$ which converts into $\bar{S}iP$; SoP having no converse has no obverted converse. Again, the obverted contrapositive of SaP is $\bar{P}a\bar{S}$ which converts into $\bar{S}i\bar{P}$, and this obverts into $\bar{S}oP$; the obverted contrapositive of SoP is $\bar{P}o\bar{S}$, and cannot therefore be converted; SiP has no obverted contrapositive; and SeP has already been dealt with. Thus only the two universals have an inverse, namely, $SaP \longrightarrow \bar{S}oP$, and $SeP \longrightarrow \bar{S}iP$. And they, of course, also have an obverted inverse, namely, $SaP \longrightarrow \bar{S}i\bar{P}$, and $SeP \longrightarrow \bar{S}o\bar{P}$. We may now tabulate the complete list of eductions. The table looks rather formidable at first sight, but is really quite easy to remember, as will be explained soon. The complete Table of Eductions is given on the next page.

COMPLETE TABLE OF EDUCTIONS.

1 Original Proposition $S—P$	2 Obverse $S—\bar{P}$	3 Converse $P—S$	4 Obverted Converse $P—\bar{S}$	5 Contrapositive $\bar{P}—S$	6 Obverted Contrapositive $\bar{P}—\bar{S}$	7 Inverse $\bar{S}—P$	8 Obverted Inverse $\bar{S}—\bar{P}$
SaP	$Se\bar{P}$	[1] PiS	$Po\bar{S}$	$\bar{P}eS$	$\bar{P}a\bar{S}$	$\bar{S}oP$	$\bar{S}i\bar{P}$
SiP	$So\bar{P}$	PiS	$Po\bar{S}$	None	None	None	None
SeP	$Sa\bar{P}$	PeS	$Pa\bar{S}$	$\bar{P}iS$	$\bar{P}o\bar{S}$	$\bar{S}iP$	$\bar{S}o\bar{P}$
SoP	$Si\bar{P}$	None	None	$\bar{P}iS$	$\bar{P}o\bar{S}$	None	None

[1] As this inverse of *SaP* distributes *P*, which was not distributed in the original, we seem to have here a violation of the general rule of formal inference stated on p. 53. The explanation is as follows. In *SaP*, P is undistributed in relation to S; but the Laws of Thought justify its distribution in relation to $\bar{S}$. The above table shows a number of cases in which $\bar{S}$ or $\bar{P}$ occurs in the conclusion (sometimes even distributed), although they are not given in the premises at all. But the Laws of Thought justify such conclusions, as is evident from the way in which the conclusions are obtained from the premises.

§ 3. *Complete Table of Eductions.*

It is unnecessary and undesirable to commit this table of eductions to memory. The only things really necessary are these: to master thoroughly the processes of obversion and of conversion; to remember that any one of the other eductions, if it can be obtained at all, can be obtained by *alternate* obversion and conversion, so that if it cannot be obtained by beginning with obversion, one should begin with conversion, and if it cannot be got then, it is not obtainable; lastly one has to remember what combination of terms each name denotes. For this last purpose the following device should be sufficient. Commit to memory:

S	P	$\bar{P}$	$\bar{S}$
Obverse	*Converse*	*Contrapositive*	*Inverse*

The symbols represent the *subjects* of the forms required. If S is the subject of the obverse, the predicate must be $\bar{P}$—not P because $S-P$ is the original. If P is subject of the converse, S will be the predicate of the converse, $\bar{S}$ of the obverted converse. Similarly with $\bar{P}$ and $\bar{S}$ as the subject of the contrapositive and of the inverse respectively.

CHAPTER VII

OTHER IMMEDIATE INFERENCES

IN addition to the formal *oppositions* and *eductions* already described, there are certain other kinds of immediate inference which may be considered now. They are (*a*) *material opposition*, (*b*) *converse relation*, or *correlation*,[1] and (*c*) *complication of terms.*

§ 1. *Material Opposition.*

In the doctrine of opposition, as treated in Chapter IV, we were concerned with various relations between propositions having precisely the same subject and the same predicate, and differing only as regards *form*, that is, in respect of quality or quantity. But relations essentially similar to those of contrariety, contradiction, sub-contrariety, and subalternation may arise from the nature of the relations between the terms employed (the form of the proposition remaining the same), or from a combination of the relations partly between the terms and partly between the forms of the propositions. In so far as these relationships (or " oppositions ") do not depend entirely on the *forms* of the propositions, but wholly or partly on their terms, they may be described as *material oppositions*, while those dealt with in Chapter IV may be called *formal oppositions*, since they depend entirely on the forms of the propositions in question, the terms being the same.

Material Contraries. (i) Two propositions may be contraries because they affirm contrary predicates

[1] Not to be confused with statistical correlation, which is something entirely different (See Ch. XX.)

of the same subject. Two terms are said to be contrary when they mean extreme opposites within the same universe of discourse, and so do not exhaust it. Thus, for example, in the universe of colour, ***white*** and ***black*** are contraries; in the realm of morality, ***saintly*** and ***wicked***; in the sphere of property, ***wealthy*** and ***destitute***, and so forth. Now two ***universal*** propositions in which contrary predicates are ***affirmed*** of the same subject are material contraries. Like formal contraries they cannot both be true, but may both be false. What is essentially the same result occurs also when the two predicates, instead of being contrary terms, are mutually exclusive but not collectively exhaustive species of the same genus, like ***white*** and ***brown***, ***red*** and ***green***, ***isosceles*** and ***scalene***, and so on.

(ii) Again, after what has been said about obversion, in Chapter V, it should be obvious that ***general*** propositions in which contradictory predicates are asserted of the same subject are contraries. For SaP is related to $Sa\bar{P}$ exactly as to its equivalent SeP; and SeP is related to $Se\bar{P}$ exactly as to its equivalent SaP.

Material Contradictories. (i) Two singular propositions in which contradictory terms are asserted of the same subject are material contradictories. Both propositions cannot be true, but one must be.

(ii) The same applies to two propositions of opposite quantity, but of the same quality in which contradictory predicates are asserted of the same subject-term. For SaP is related to $Si\bar{P}$ exactly as to its equivalent SoP; and SeP is related to $So\bar{P}$ in the same way as to its equivalent SiP.

Material Sub-Contraries. Obviously SiP is related

to $Si\bar{P}$ in the same way as to SoP; and SoP is related to $So\bar{P}$ in the same way as to SiP.

Material Subalternation. Suppose we have a proposition of the type SaP in which S is a general term including two or more sub-classes, say, S_1, S_2, etc. then SaP obviously implies S_1aP, S_2aP, etc., while S_1aP, or S_2aP, etc., does not imply SaP. For instance, *Scandinavians are Europeans* implies *Danes are Europeans*; *Parallelograms have their opposite sides equal* implies *Rectangles have their opposite sides equal.* Similarly with SeP in relation to S_1eP, S_2eP, etc. For example, *Scandinavians are not of Mediterranean race* implies *Swedes are not of Mediterranean race.* Again, suppose that P is a sub-class (or species) of a more general term (or genus), say Q, then SaP implies SaQ because the attributes implied by P (the specific term) include the attributes implied by Q (the generic term). Thus *Cornishmen are Englishmen* implies *Cornishmen are British.* On the other hand SaQ does not imply SaP. The relation of SaP to S_1aP, or S_2aP, etc., and of SeP to S_1eP, or S_2eP, etc., also of SaP to SaQ, is that of subalternation, or that of *subalternant* (the proposition which implies another without being implied by it) to that of *subalternate* (the proposition which is implied by another without implying it). But the relation between SeP and SeQ is the reverse of that between SaP and SaQ—SeQ implies SeP, not *vice versa.* So, for instance, *No Indians are Englishmen* does not imply *No Indians are British*, but *No Egyptians are British* would imply *No Egyptians are English.*

§ 2. *Immediate Inference by Converse Relation.*

In some propositions the predicate (P) asserts some definite relation (say, R) in which the subject (S)

stands to a certain term (say, Q). In such cases we can deal with the predicate (*P*) as a whole, and so obtain the ordinary converse of the proposition (*P—S*), etc., or we may analyse the predicate into its components (*R*, Q) and infer that since *S* stands to Q in the relation *R*, Q must stand to *S* in the converse relation (or correlation) *Я*, where *R* and *Я* represent a pair of correlative terms, that is, terms which are specially given to draw attention to a definite relationship between certain objects, like ***north*** and ***south*** ***right*** and ***left***, ***teacher*** and ***pupil***, etc. Take the proposition *S* ***is due north*** (*R*) ***of*** Q. The ordinary converse would be ***Some place due north of*** Q ***is*** *S* (***PiS***). But by converse relation it also implies Q ***is due south of*** *S* (Q ***is*** *ЯS*), which may be called its ***correlative proposition.*** Similarly the proposition ***University professors are teachers of University students*** implies, as its ***converse, Some teachers of University students are University professors,*** but its ***correlative*** is ***University students are pupils of University professors.*** Inference by converse relation (or correlation) is very common in ordinary life. It frequently happens that we want to know the relation of Q to *S*, but, if *S* is the more important of the two, the works of reference we consult will give us information about the relation of *S* to Q. This may serve our purpose just as well, because we can formulate for ourselves the required correlative.[1]

[1] That the mutual implication of correlative propositions is not always appreciated appears from the following story. A lady, it is related, told her shoemaker that one of her feet was bigger than the other. "Quite the contrary, madam," said the gallant shoemaker, "I find that one of your feet is smaller than the other."

§ 3. *Immediate Inference by Complication of Terms.*

Given the proposition *S is P* it is possible and sometimes convenient to add a third term to both *S* and *P*, say *D*, and so obtain the inference *DS is DP* the terms of which are more complex than those of the original proposition. For example, the proposition *Triangles are figures* implies the propositions *Plane triangles are plane figures, Equilateral triangles are equilateral figures, Spherical triangles are spherical figures, Combinations of triangles are combinations of figures, The study of triangles is the study of figures, The symbolic use of triangles is the symbolic use of figures*, and so on. Some distinguish two varieties of this kind of immediate inference, namely, that *by added determinants* and that *by complex conception*, according as *D* is a determin*ant* of *S* and *P* (that is, restricts or qualifies them), or a determin*atum* of *S* and *P* (that is, is restricted or qualified by them). The first three of the above examples illustrate immediate inference by added determinants; the last three illustrate immediate inference by complex conception. But it is simpler to treat them both under a common designation, for they are essentially alike.

The validity of immediate inference by complication of terms depends on two conditions: (i) The determinant or determinatum (*D*) must be relevant to the terms of the original proposition (*S* and *P*); (ii) it must have precisely the same meaning in conjunction with both terms. (i) If it is not relevant, the result will be sheer nonsense. (ii) If it has not the same meaning in the two cases it is not really the same determinant or determinatum

This form of inference, where it is employed, is rarely set out explicitly. The "given proposition"

(*S is P*) is tacitly assumed rather than clearly formulated, and the conclusion usually takes the form of substituting one term for another. One speaks of a strong *poison*, or of a dose of *poison*, instead of a strong *arsenic*, or a dose of *arsenic*, for instance, on the implicit ground that *arsenic is a poison*, *S is P*, therefore *DS is DP*. But mistakes are all the more apt to arise when the ground of the inference is not stated explicitly. The chief source of mistakes is to be found in what may be called the *relativity* of certain terms expressing quantity or degree. Some terms may be said to express such values absolutely—for example, *an English mile*, 100° *Centigrade*, £100 *sterling*, and so on. There are other terms which express such values not absolutely, but relatively—for instance, *far*, *near*, *hot*, *warm*, *rich*, *poor*, *big*, *small*, etc. Such relative terms of measure are apt to suggest, or even imply, rather different values in different contexts, and so involve occasionally a breach of the second condition stated above. Thus, for instance, it would be misleading to describe even the smallest elephant as "a tiny animal" merely on the ground that *an elephant is an animal*; or to describe ice-pudding as "a warm dish" merely because *it is a dish* and it is not as cold as it should be; or to describe a dock-labourer in receipt of a considerable unemployment dole as "a rich man," merely because *a dock-labourer is a man*, and he is comparatively well off for his station and under the circumstances Similarly it would be false to describe the views of the majority of barristers, or of head masters, as the views of the majority of lawyers, or of teachers, respectively, merely because *barristers are lawyers*, and *head masters are teachers*.

CHAPTER VIII

MEDIATE INFERENCE

§ 1. *The General Character of Mediate Inference.*

So far we have considered the inferences that may be drawn from single categorical propositions. We must determine next what inferences may be drawn from two or more of them jointly, over and above the immediate inferences implied by each of them separately. It is obvious that two or more propositions involving entirely different terms cannot between them imply anything more than the sum of their separate implications. But where two propositions have a term in common, then something may be inferable from the two together which could not be inferred from either separately, and which is not merely the sum of their separate implications. This common term may mediate between the other terms of the two propositions so as to establish a relationship between them that could not be inferred from either proposition alone. That is of the essence of mediate inference—given the relationship of each of two terms to the same third term, it is possible under certain conditions to infer their relationship to each other. Symbolically, if we know how *S* is related to *M*, and how *P* is related to *M*, it may be possible to infer how *S* is related to *P*.

A story related of an incident which is said to have happened at a reception given by a French Countess may serve as an illustration of mediate inference. Among the early callers was a certain Cardinal, in conversation with whom his hostess remarked sympathetically on the wide and varied experience he must have had. The Cardinal assented, adding that

he had, in fact, started rather badly, for the very first person to confess to him had confessed a murder! Some time afterwards, while the Cardinal was conversing with some one at the far end of the *salon*, a well-known French Count called, and the hostess, after chatting with him a while, suggested that she would like to introduce him to the Cardinal. The Count replied that no introduction was necessary as he had known His Eminence many years. "In fact," added the Count, "I was the very first person to confess to him, and (he added with a twinkle in his eye) let me assure you, Countess, that my confession did surprise him!" The feeling of the hostess may easily be imagined.

§ 2. *Mediate Inference with a Singular Middle Term.*

In the foregoing illustration the middle term is singular (that is, denotes an individual object), and the relation between the terms is that of identity. When that is the case mediate inference is easy and obvious. The identity of "the Count" with "the first person who confessed to the Cardinal," and of this person with "a person who confessed a murder," implies the identity of "the Count" with the "person who confessed a murder." It is evident that if two terms (S and P) are each identical with the same singular third (or middle) term (M), then they must be identical with each other. M *is* P, S *is* M, $\therefore$ S *is* P. S being identical with M, they are really different names of the same thing, so that what is called M may also be called S. Now, if it were possible in this case for S not to be P, then the same thing (called indifferently S or M) would be P, according to the first premise, and would not be P, if the suggested

conclusion were rejected. But that would violate the Law of Contradiction.

The conclusion, *S is P*, is therefore valid. So far only affirmative propositions have been considered as the premises of mediate inferences. Is it possible to draw mediate inferences also from negative premises? A distinction must be drawn between cases in which both premises are negative, and those in which one premise is negative, and the other is affirmative.

When both premises are genuinely negative, and not merely the obverse equivalents of affirmative propositions, then they do not warrant any mediate conclusion. From *S is not M* and *M is not P*, no inference can be drawn with reference to *S* and *P*. There is no real mediation in such cases, for we are not really told how *S* and *P* ***are*** related to *M*, only that they are ***not*** related to it; and their common difference from *M* in some respects is compatible with the relation either of identity or of difference between *S* and *P*. Cases like *S is not* $\overline{M}$ and *M is not* $\overline{P}$, $\therefore$ *S is P*, are no real exceptions because the premises are merely obverse forms of *S is M* and *M is P*, and since *M* is assumed to be singular, the conclusion is legitimate. But if both premises are genuinely negative, that is, genuine negations, then no mediate inference is justified.

On the other hand, if one premise is affirmative, and the other is negative, then it is sometimes legitimate to infer a conclusion, but not always. Let the relationship asserted in the affirmative proposition be that of identity, then the negative premise will in that case deny identity, or, what is the same thing, assert difference. *S* and *P* will in that case be related in contradictory ways to the same third term, *M* (which is still assumed to be a singular term), and must

consequently be different from each other in some respect, that is, S *is not* P. Thus S *is* M and M *is not* P imply S *is not* P; and S *is not* M and M *is* P also imply S *is not* P. In the first example S and M are identified in the first premise, and so S may be substituted for M in the second premise, and then the Law of Contradiction would be violated if the conclusion were S *is* P. Similarly, in the second example, M is identified with P in the second premise, and so P may be substituted for M in the first premise, and then the Law of Contradiction would be violated if the conclusion were S *is* P.

§ 3. *Identity and other Transitive Relations.*

In the foregoing account of mediate inference with singular middle terms the premises were all such as expressed relations of identity and difference. But some of the results arrived at are applicable also to all other transitive relations. A transitive relation is such that if it holds good between one term and a second term, and likewise between the second term and a third term, then it will also hold good between the first term and the third term. Identity is one such relation—if S is M and M is P, then S is P, as just explained. Equality is another transitive relation—if $S = M$, and $M = P$, then $S = P$. The relations of "greater than" and "less than" are also transitive—if $S > M$ and $M > P$, then $S > P$. But there is no general way of indicating how transitive relations may infallibly be distinguished from others, and care has to be exercised. For example, while "brother of," "sister of," "ancestor of" are transitive relations, "parent of," "uncle of," "brother-in-law of" are not transitive relations. So long,

however, as the relation is transitive, and the middle term is singular, the two propositional forms SrM, MrP will imply SrP—where r stands for the assertion of the relation in question.

When both premises are affirmative there is, then, no essential difference between mediate inference from singular premises which express relations of identity and those which express other transitive relations.

Similarly, when both premises are negative, no valid conclusion can be drawn in either case. From *S is not r M* and *M is not r P* nothing can be inferred with regard to the relation between *S* and P. Their common difference from *M* may be compatible with all sorts of relations between *S* and *P*.

It is entirely different when one premise is affirmative and the other is negative. With some kinds of transitive relations, such as equality or parallelism, for instance, the case is just like that of identity, and a negative conclusion follows. Thus, for example, if *S* and *P* are one of them equal to *M* and the other not, then *S* is not equal to *P*. But it is different with other relations, even transitive relations. From *S is greater than M* and *M is not greater than P* no inference can be drawn, while *S is greater than M* and *P is not greater than M* imply that *S is greater than P*.

Unfortunately no general rules can be laid down for propositions which do not express relations of identity, or which cannot be so interpreted as to express relations of identity (or difference, when the propositions are negative).

The relation of identity (and its negation, difference) is, however, a very comprehensive one, and other relations are sometimes reducible to it. Take, for

instance, the relation of equality. It is only a more concrete way of expressing a more abstract identity of some quantity or other. For example, let S, M and P stand for lines. Then $S = M, M = P, \therefore S = P$ may be restated just as accurately, or even more so, in terms of identity. Thus: *The length of S is the length of M, the length of M is the length of P, ∴ the length of S is the length of P.* Similarly, the argument *S is as rich as M*, and *M is as rich as P, ∴ S is as rich as P*, may be restated in the following statements of identity. *The value of the possessions of S is the value of the possessions of M, The value of the possessions of M is the value of the possessions of P, ∴ The value of the possessions of S is the value of the possessions of P.* (Needless to say, it is not suggested that S, M and P are identical in these cases, in which S, M and P are not the whole predicates, but only parts of them.) With a little ingenuity other relations can likewise be expressed in terms of identity; but it would take us too far afield to consider them here. Enough that a great number of the premises with the implications of which one is mostly concerned either are, or can be, expressed in terms of identity and difference.

§ 4. *Dovetail Relations.*

Besides transitive relations there are also certain other relations which frequently occur in mediate inference. These may be called *dovetail* relationships, for the following reason. In the case of mediate inference involving transitive relations what usually happens is that the premises attest that the major term and the minor term are each related in the same kind of way to the same middle term, and the conclu-

sion states that the same relationship therefore holds good between the major and minor terms themselves. The cases in which one premise is negative are not essentially different—they also refer to the same relationship throughout, which is affirmed in one premise, denied in the other, and usually denied in the conclusion. It is different with the cases now under consideration. In these cases one premise may assert one kind of relationship, the other a different relationship, and in the conclusion yet a third relationship is obtained by dovetailing the other two relationships, in the light of our knowledge of the system of relationships involved. An example or two will make this clear.

(1) *M is the brother of P,*
S is the son of M,
∴ *S is the nephew of P.*

(2) *M is due north of P,*
S is due west of M,
∴ *S is north-west of P.*

It should be realized that inferences involving other relations than those of identity and difference (or relations reducible to these) cannot be made safely without a knowledge of the actual system of relationships involved in each case. They are not so formal as those involving only relations of identity and difference. Hence they are usually ignored in Formal Logic. But the mediate inference involved is essentially of the same character throughout, and the systems of relationships mostly concerned are such as intelligent people are generally conversant with.

§ 5. *Rules of Mediate Inference with a Singular Middle Term.*

The results reached so far, with regard to mediate inference from particular cases, may be summarized as follows:

(1) If both premises are irreducibly negative (that is, are not merely the obverses of affirmative propositions), then no mediate inference is justified.

(2) If the middle term is singular, and both premises are affirmative, and express a relation of identity, or some other transitive relation, then it is permissible to draw an affirmative conclusion with the other two terms of the premises. Symbolically: *S is M, M is P*, ∴ *S is P*, or, more generally, *SrM, MrP*, ∴ *SrP*, where *r* represents the affirmation of any transitive relation.

(3) If the middle term is singular, and the premises are one affirmative and one negative, and express a relation of identity and difference repectively, then a negative conclusion is inferable. Symbolically: *S is M, M is not P*, ∴ *S is not P*; or *S is not M, M is P*, ∴ *S is not P*.

Conversely, to draw a negative inference one premise must be affirmative, and one negative. This follows from (1) and (2).

In this case, unlike (1), it is not possible to extend the formulæ so as to include *all* transitive relations; but some such relations, for instance, those of equality and parallelism, can be treated in the same way as identity

CHAPTER IX

MEDIATE INFERENCE WITH A GENERAL MIDDLE TERM

§ 1. *Complications Arising when the Middle Term is General.*

(*a*) We may now consider the conditions of mediate inference when the middle term is not a singular term but general (or a class name). In this case certain complications arise. So long as the middle term (*M*) is unambiguous and singular the main feature of mediate inference is obvious, and its principal condition is satisfied, namely, that it is really the same third term with which the other two terms (*S* and *P*) are compared. It is obvious that if the two premises involve entirely different terms, say *S* and *M*, in one case, and *Q* and *P*, in the other, then no inference can be drawn about *S* and *P*. Now the mere fact that the same *general name* occurs in two propositions does not necessarily mean that the same *things* are referred to. The members of the class referred to in one proposition may be quite distinct from those referred to in the other proposition, and so there may be no real mediating link. For example, suppose we have the premises, *All Danes are Scandinavians*, and *Some Scandinavians are Norwegians*. It is obvious that the Scandinavians mentioned in the first proposition are not the same as those mentioned in the second Danes and Norwegians are consequently not identified with the *same* third term, and it would therefore be erroneous to identify them with one another. Under what circumstances, then, when the middle term is general, can we be sure that the two premises, in both of which

it occurs, really have the same middle term? The answer is, when the middle term is distributed in at least one of the two premises. For, if one premise refers to the whole of the class denoted by the middle term, then whatever part of that class the other premise may refer to, the two premises are bound to have something really in common. For example, the two propositions *All Danes are Scandinavians*, and *All Scandinavians are Europeans*, do imply that *All Danes are Europeans.* Here the middle term (Scandinavians) is distributed in the second premise, though not in the first. Consequently the Scandinavians referred to in the first premise are identical with some of those referred to in the second, and the fact that the Danes and some Europeans are each identified with the same Scandinavians, implies the identity of the Danes with some Europeans. When the middle term is singular it is not necessary to say anything about its distribution, because it is inevitably distributed, as it denotes an individual object and cannot denote less. But the requirement that the middle term should be distributed in at least one of the premises really is a rule of all mediate inference so long as numbers or numerical proportions form no part of the data or premises

(*b*) Again, so long as the middle term is singular both premises must be singular, and therefore the conclusion is usually singular.[1] But when the middle term is general, the premises may be general or particular as

[1] For the middle term must be either subject or predicate in the premises. If it is subject and singular the proposition is singular (by definition). If it is predicate and singular, it can only be predicated of a singular subject, and then the proposition is again singular.

well as singular. And then the question arises as to what kind of a conclusion can be drawn, whether it can be universal or must be particular. Here applies obviously the general rule of formal inference already stated in an earlier chapter, namely, that the conclusion must not distribute a term which was not distributed in its premise. If, therefore, the term which becomes the subject of the conclusion (*the minor term*, as it is called) is not distributed in its premise (*the minor premise*) then the conclusion (if any) can only be particular.

(*c*) Once more, so long as a term is singular it is always distributed, no matter whether it is the subject or the predicate of a proposition, and no matter if it is predicate of an affirmative or of a negative proposition. But when the terms of a mediate argument are general, it is quite different. In such an affirmative proposition the predicate is never distributed; in a negative proposition it always is. Now if a conclusion is negative its predicate will be distributed; but it must not be distributed unless that term (*the major term*, it is called) is distributed in its premise (*the major premise*). This means that a negative conclusion is invalid unless the major term is distributed in the major premise.

(*d*) Again, when considering mediate inference with a singular middle term, it was shown that when both premises are really negative then no mediate inference is valid. This rule holds good also when the middle term is general. The reasons are the same.

(*e*) Similarly, in the case of mediate inference with a general middle term, as in the case of mediate inference with a singular middle term, and for the same reasons, when both premises are affirmative the con-

clusion can only be affirmative. And in this case we are not confined to the relation of identity, but can deal in the same way with all transitive relations.

(*f*) Lastly, if one premise is affirmative and the other is negative, only a negative conclusion can be inferred, when the middle term is general as when it is singular, and for the same reasons. And conversely, a mediate negative inference can only be drawn when one premise is affirmative and the other negative. This follows from (*d*) and (*e*) above. If two negative premises warrant no inference whatever, and two affirmative premises can only justify an affirmative conclusion, then a negative inference can only be drawn from two premises one of which is affirmative and the other negative.

§ 2. *General Rules of Mediate Inference.*

It will have been seen that mediate inference when the middle term is singular is simpler in various ways than when the middle term is general, because the whole question of the distribution of terms does not arise, singular terms being distributed in any case. At the same time the conditions concerning the distribution of terms when the middle term is general apply also to the case when the middle term is singular, even if they call for no special attention, because they are inevitably satisfied under the circumstances. The more comprehensive conditions (or rules or norms) of mediate inference when the middle term is general may, therefore, be formulated as the general rules of mediate inference whether the middle term is general or singular. We may accordingly set out completely the general conditions or rules of mediate inference as follows:

1. There are three propositions—two premises and a conclusion.

2. There are three distinct terms, one of which (the middle term) occurs in both premises, each of the other two only in one of the premises, but also in the conclusion—the term which is the subject of the conclusion is called the minor term, and the term which is the predicate of the conclusion is called the major term.

3. The middle term must be distributed once at least.

4. No term may be distributed in the conclusion if it is not distributed in its premise—in other words, the conclusion may be universal only if the minor term is distributed in its premise, and negative only if the major term is distributed in its premise.

5. No conclusion may be inferred from two irreducibly negative premises—or, in other words, one premise at least must be affirmative.

6. If both premises are affirmative the conclusion can only be affirmative.

7. If one premise is negative the conclusion can only be negative, and conversely if the conclusion is to be negative one of the premises must be negative.

These general rules of mediate inference (or general rules of the syllogism,[1] as they are more usually called) could of course, be reduced in number, if our object were to formulate only principal rules, not derivative ones. In fact, we arrived at them without relying on much more than (1) the need of a real middle term, or mediating link, (2) the Law of Contradiction, and (3) the rule of all formal inference that the con-

[1] This will be explained in the next chapter

clusion must not distribute a term undistributed in the premises. But the explicit formulation of all the important rules is much more helpful.

The foregoing general rules are quite sufficient to determine the nature or validity of a mediate inference in relation to given premises. What follows is only an elaboration of derivative details, which may be useful and interesting, but are not of the same importance. Once the general rules are mastered there should be no difficulty in the elaboration of the consequent details.

The three most comprehensive consequences which follow from the general rules of mediate inference are the following three corollaries:

(*a*) If both premises are particular, and do not specify certain numbers or proportions, no conclusion can be drawn.[1]

(*b*) If one premise is particular the conclusion (if any) can only be particular.

(*c*) If the major premise is particular, and the minor premise is negative, no conclusion can be inferred.

The reason for all the corollaries is to be found in the fact that not enough terms are distributed, under the conditions supposed, to warrant anything else than is prescribed by the corollaries. Let us consider them each separately.

[1] Two particular premises such as

Most M's are P,
Most M's are S,

do warrant the conclusion

Some S's are P,

because the proportions of *M* are such as to ensure a real middle term, or common element, in the two premises.

(*a*) Both premises are assumed to be particular. There are only two possibilities which need be considered: (i) Either both premises are affirmative, or (ii) one premise is affirmative, and the other is negative. If (i), then no term, not even the middle term, is distributed, so there can be no conclusion. If (ii), only one term will be distributed, namely, the predicate of the negative premise. But in this case at least two terms should be distributed to justify a conclusion, namely, the middle term and the major term. Therefore no conclusion.

(*b*) One premise is assumed to be particular. If there is to be a conclusion at all, the other premise must be universal (*a*). Now there are only two possibilities which need be considered. (i) Either both premises are affirmative, or (ii) one premise is affirmative, and the other is negative. If (i), then the two premises will only distribute one term between them, namely, the subject of the universal premise. Now to warrant a conclusion the distributed term must be the middle term, so the minor term is not distributed in its premise, and must not be distributed in the conclusion, that is, the conclusion can only be particular. If (ii), two terms will be distributed in the premises, namely, the subject of the universal premise, and the predicate of the negative premise. The inference in this case can only be negative, and so at least two terms must be distributed, namely, the middle term and the major term. The minor term, in that case, will not be distributed, and the conclusion can only be particular.

(*c*) Here the minor premise is assumed to be negative, and the major premise particular. If, therefore, there is to be any chance of a valid inference, the

major premise must be affirmative. But if the major premise is particular affirmative, it distributes neither of its terms. So the major term is not distributed. Since, however, one premise is negative, the conclusion, if any, will have to be negative, and distribute the major term. Therefore there can be no conclusion in this case.

CHAPTER X

DEDUCTION AND SYLLOGISM

§ 1. *Mediate, Deductive, and Syllogistic Inference.*

Deduction, or deductive inference, is usually defined as inference from a general proposition, or from general propositions, or as the application of laws (or rules) to relevant cases. It is usually contrasted with induction, which is inference from particulars. And syllogism is usually described at once as mediate inference and as deductive inference, and also as confined to propositions which express the relation of attribute to substance. If this conception were strictly adhered to, the term syllogism would coincide with only those mediate inferences which have a general middle term, and which only deal with relations of identity and difference, or with relations which are reducible to those of identity or of difference. In practice, however, the treatment of the subject is not consistent in books on Logic, and some cases of mediate inference with a singular middle term, and without a general premise, are usually included among syllogisms. The reason for the inconsistency is to be found in an excessive leaning on the Aristotelian *dictum de omni et nullo* (or some similar formula) as the basis of syllogistic inference. Now, it is convenient to use the term *Deduction* in the above mentioned sense, which is different from that of *mediate inference,* in its widest sense; but there seems to be no good reason for using the term *syllogism* in any other sense than that of mediate inference. After all, *syllogism* only means

putting two and two together, and that is just what is done in *all* mediate inference. Except out of respect for tradition, the term syllogism might be dropped altogether, and be replaced by *mediate inference.* But there is no point in being unnecessarily iconoclastic so long as serious disadvantages, such as those of ambiguity, can be avoided. The term *Syllogism* also has the advantage of greater brevity than the term *mediate inference.* In this book, accordingly, the term *syllogism* will be understood as synonymous with *mediate inference*—a wider meaning than it has in other books on Logic. The term *Deduction* should also be used more consistently than is usually the case. Instead of confining it, as is nearly always done, to syllogistic inference with a general premise, it should be used to include also such inferences as immediate inference from an *A* to an *I* proposition, or from an *E* to an *O* proposition (usually called immediate inference by subalternation). According to the usage of terms here suggested and employed, an inference may be both deductive and mediate (or syllogistic), or it may be mediate without being deductive, or deductive without being mediate. For example, the argument $X = Y$, $Y = Z$, $\therefore X = Z$, is mediate, but not deductive; the argument, *All radii of the same circle are equal, ∴ radii AB, AC, AD of circle BCD are equal,* is deductive, but not mediate; lastly, the argument *All who profit from war-preparations are prone to believe in the inevitableness of warfare. Manufacturers of arms and armaments profit from war-preparations, ∴ they are prone to believe in the inevitableness of warfare,* is both deductive and mediate.

We shall now proceed to consider the varieties of mediate inference, or syllogism.

§ 2. *Figure and Mood of Syllogisms.*

Every syllogism, as we have seen, has two premises, a major premise and a minor premise. The major premise contains the middle term and the major term (M and P); the minor premise contains the middle term and the minor term (M and S). Now in each premise either of its terms might be subject and the other predicate. In other words the major premise might be either $M-P$, or $P-M$, and the minor premise might similarly be either $M-S$, or $S-M$. Consequently, there are four possibilities with regard to the arrangement of the terms in a syllogism:

Major premise:	$M—P$,	$P—M$,	$M—P$,	$P—M$.
Minor premise:	$S—M$,	$S—M$,	$M—S$,	$M—S$.

The conclusion in each case is assumed to be $S—P$, the names of the terms and of the premises being based on this assumption. Now these differences in the arrangements of the terms of a syllogism are called differences of *Figure.* The above four arrangements are obviously the only possible ones, and they are known respectively as the First, Second, Third, and Fourth Figure. The First Figure is the one most commonly used. It is the most natural Figure, as the term which is subject of the conclusion is also subject of its premise, and the term which is predicate in the conclusion is also predicate in its premise. In the Fourth Figure the arrangement of terms is just the opposite to that in the First Figure. In the Second Figure the middle term is predicate in both premises. In the Third Figure the middle term is subject in both premises.

It will be seen that each Figure is a kind of scheme

of syllogisms. With the same arrangement of terms in the premises, the premises may obviously vary in quality and quantity. These differences of quality and quantity were abstracted from, when differences of Figure were described. But, of course, there can be no proposition without some quality and quantity. Now differences in the quality and quantity of the propositions constituting a syllogism are called differences of *Mood.* And *prima facie* each Figure may have a number of different moods. For example, the syllogistic types

MaP	*MeP*
SaM	*SaM*
∴ *SaP*	∴ *SeP*

are different moods of Figure I. The problem thus arises as to how many valid moods there are in each Figure, and what they are. This problem is known as that of the determination of the valid moods of the syllogism, and the problem is solved with the aid of the general rules of the syllogism and the corollaries already stated and explained.

§ 3. *The Determination of the Valid Moods.*

The general rules of the syllogism and the corollaries state what kind of premises, as regards quality and quantity, can or cannot justify a conclusion. Our best plan will therefore be, in the first instance, to abstract from differences of Figure, to consider the various conceivable combinations of premises having regard to their quality and quantity only, and to find out which of them are disqualified, by the general rules and corollaries, from yielding a valid conclusion in any case, that is, in any Figure. Now as regards

quality and quantity there are four propositional forms, *A*, *I*, *E*, and *O*. *Prima facie* any one of these might serve as major premise, or as minor premise, or both. There are consequently sixteen conceivable combinations of premises. Putting the major premise first, and the minor premise second, the sixteen possible combinations of premises are as follows:

AA, *AI*, *AE*, *AO*,
IA, *II*, *IE*, *IO*,
EA, *EI*, *EE*, *EO*,
OA, *OI*, *OE*, *OO*.

Of these sixteen combinations, eight cannot justify any conclusion. They are: *EE*, *EO*, *OE*, *OO* (Rule 5), *II*, *IO*, *OI* (Corollary (*a*)), and *IE* (Corollary (*c*)).

Thus only eight of the conceivable combinations of premises are likely to justify a conclusion in some Figure or other. They are *AA*, *AI*, *AE*, *AO*, *IA*, *EA*, *EI*, *OA*. But we cannot say what conclusion, or in which Figure, without examining each of the surviving combinations of premises in relation to each of the Figures. The reason for this should be obvious. Each combination of premises means something different in each figure, and consequently it may imply one kind of conclusion in one Figure, another in another, and perhaps no conclusion at all in a third. For example, *AA* means *MaP*, *SaM* in Fig. I and implies *SaP*; it means *PaM*, *SaM* in Fig. II and implies nothing at all here, because the middle term is not distributed. So we must examine each of the eight surviving combinations of premises in each of the four Figures:

***AA*, *AI*, *AE*, *AO*, *IA*, *EA*, *EI*, *OA*.**

FIGURE I. Scheme:

$$\begin{array}{l} M - P \\ S - M \\ \therefore S - P. \end{array}$$

Valid moods[1]: *AAA*, *AII*, *EAE*, *EIO*.

The premises *AE* and *AO* justify no conclusion because the conclusion (if any) could only be negative, and the major term is not distributed. *IA* and *OA* fail likewise because the middle term is not distributed.

FIGURE II Scheme:

$$\begin{array}{l} P - M \\ S - M \\ \therefore S - P. \end{array}$$

Valid moods: *AEE*, *AOO*, *EAE*, *EIO*.

[1] The procedure in determining the valid moods in each Figure may be briefly summarized as follows. Write down the premises in the form which they assume in the Figure under consideration (e.g. Fig. I.

$$\begin{array}{l} MaP \\ SaM \end{array}$$

or Fig. II

$$\begin{array}{l} PaM \\ SaM \end{array}$$

and so on).

Then see if *M* is distributed in one of the premises. If not, then no conclusion follows.

If *M* is distributed, and if both premises are affirmative, then an affirmative conclusion follows and the conclusion is universal if *S* is distributed in the minor premise, particular if it is not. But if one premise is negative, then not only must *M* be distributed, but also *P*. If both *M* and *P* are distributed in the premises, and only then, a negative conclusion follows, and the conclusion is universal if *S* is distributed in its premise, particular if it is not.

The premises *AA*, *AI*, *IA* yielded no conclusion because the middle term is not distributed; and *OA* because the major term is not distributed, while the conclusion (if any) would have to be negative

FIGURE III. Scheme:

$$\begin{array}{l} M-P \\ M-S \\ \therefore S-P. \end{array}$$

Valid moods: *AAI*, *AII*, *IAI*, *EAO*, *EIO*, *OAO*.

The premises *AE* and *AO* justify no conclusion because the major term is not distributed, while the conclusion (if any) could only be negative.

FIGURE IV. Scheme:

$$\begin{array}{l} P-M \\ M-S \\ \therefore S-P. \end{array}$$

Valid moods: *AAI*, *AEE*, *IAI*, *EAO*, *EIO*.

The premises *AI* and *AO* yield no conclusion because the middle term is not distributed; and *OA* because the major term is not distributed, while the conclusion (if any) could only be negative.

§ 4. *Special Rules of Each Figure.*

If the valid moods of each Figure are examined in turn, it will be found that each Figure has its peculiarities and that these peculiarities are the logical result of the application of the general rules of the syllogism to the special arrangement of the terms in each Figure.

These peculiarities constitute what are known as the *special rules of each Figure*, which may be considered now.

(*a*) *Special Rules of Figure I.* A glance at the valid moods of Figure I shows that

(i) the minor premise is always affirmative, and
(ii) the major premise is always universal.

The reason is that in this Figure ($M-P$, $S-M$, $\therefore S-P$) a negative minor premise, involving an affirmative major premise, would necessitate an undistributed major term (unless it happened to be singular) and so could not warrant a negative conclusion, which is the only one possible if one premise is negative. The minor premise must therefore be affirmative, and so does not distribute the middle term (unless it is singular), which must consequently be distributed in the major premise, and this can only be done if the major premise is universal.

(*b*) *Special Rules of Figure II.* It will be noticed that in all the valid moods of this Figure

(i) one premise is negative,
(ii) the major premise is universal.

The reason is as follows: In this Figure ($P-M$, $S-M$, $\therefore S-P$) the middle term is predicate in both premises, and (unless it is singular) can therefore only be distributed if one premise is negative. Now a negative premise necessitates a negative conclusion (if any), and this requires the distribution of the major premise, which must consequently be universal.

(*c*) *Special Rules for Figure III.* In all the valid moods of this Figure

(i) the minor premise is affirmative,
(ii) the conclusion is particular

The reason for (i) is the same as in the case of Figure I, namely, to prevent illicit distribution of the major term in the conclusion. But the minor premise being affirmative it cannot distribute the minor term in this Figure ($M-P$, $M-S$, $\therefore S-P$), unless it is a singular term Therefore the conclusion cannot be general, only particular (or singular).

(*d*) *Special Rules of Figure IV.* The peculiarities of the moods of this Figure are not so obvious, but they will be seen to be the following:

(i) when either premise is negative the major is universal,
(ii) when the major premise is affirmative the minor is universal, and
(iii) when the minor premise is affirmative the conclusion is particular.

The reason for (i) is obvious if one looks at the scheme of this Figure ($P-M$, $M-S$, $\therefore S-P$). A negative premise necessitates a negative conclusion, if any, and this requires the distribution of the major term in the major premise, which must accordingly be universal (compare the special rules for Figure II). Again, (ii) if the major premise is affirmative it does not distribute the middle term (unless it is singular), which must consequently be distributed in the minor premise, and so the minor premise must be universal. Lastly, the reason for (iii) is the same as in Figure III, namely, the minor term being undistributed in its affirmative premise (unless it is singular) must not be distributed in the conclusion.

it only remains to point out again that the special rules of the Figures are simply derived from the general rules of the syllogism, and involve no new principles. When testing the validity of any syllogism it is quite unnecessary to apply the special rules of the Figure to which it belongs, or indeed to trouble about its Figure at all; the general rules of the syllogism can be applied to it directly.

§ 5. *Quantitative Deduction.*

The kind of deductive reasoning considered in the preceding sections may be described as *qualitative deduction.* It consists in the application of general propositions to certain cases or classes of cases which are of a certain kind, namely, of the kind referred to in the general propositions applied. We argue, for example, that *the clouds must be subject to gravitation,* because *all material bodies are subject to gravitation*; or we conclude that *Jones is liable to pay income tax,* because *all people who are domiciled in this country, and whose income exceeds a certain minimum scale, are liable to pay income tax*; and so on. These examples are merely qualitative, not quantitative—no reference is made to the quantity of the gravitation or to the amount of income tax in the above examples. In a great many cases, however, both in science and in daily life, the general propositions, principles, or regulations which are applied have a quantitative form, and the proper application of them involves reckoning as well as reasoning. Not only must we see to it that the cases to which they are applied are relevant cases, we must also make our calculations correct to the degree required. The following are a few simple examples of such quantitative deduction. (1) Accord-

ing to the Law of Gravitation, two masses M_1 and M_2 whose distance from each other is d, are pulled together each with a force GM_1M_2/d^2, where G is the gravitation constant for all kinds of matter (approximately $6 \cdot 66 \times 10^{-8}$). Now the mass of Jupiter is 315 times that of the earth, and his mean distance from the sun $5 \cdot 2$ times that of the earth. Therefore the sun's attraction on Jupiter is $315/5 \cdot 2^2$ times that on the earth, that is, $11 \cdot 6$ times as great. (2) According to Boyle's Law, the volume of a gas, at a constant temperature, varies inversely with the pressure. Consequently if a column of air confined in a Torricellian tube at a pressure of one atmosphere is 50 cm. high, it will be reduced to 25 cm. when the pressure is increased to that of two atmospheres. (3) The coefficient of expansion of air under constant pressure is approximately $1/268 \cdot 5$. Therefore 90 c.c. of air at 0° C. if heated to 30° C. will increase to $90 (1 + 30/268 \cdot 5)$ c.c., that is, approximately 100 c c. (4) According to Ohm's Law, the strength of an electric current (C) passing through a conductor is directly proportional to the difference of potential (E) between the two ends of the conductor, and inversely proportional to the resistance (R) of the conductor. (Usually C is given in ampères, E in volts, R in ohms.) If, therefore, the filament of my electric lamp has a resistance of approximately 900 ohms, and the potential of my electrical supply is 220 volts, then the strength of the current passing through the filament of the lamp must be 220/900, that is, ¼ ampère approximately. (5) According to the laws relating to British income tax, no tax is paid on the first sixth part of earned income (within certain limits) nor on the next £135. There are also various reliefs for dependents, etc. The

rest of the earned income is subject to a tax of 10 per cent. on the first £225, and 20 per cent. on the rest (within certain limits). If, therefore, Jones has no dependents, is not entitled to any special reliefs, and earns an annual income of £900, he must pay income tax on (£900 − £150 − £135 =) £615, namely, 10 per cent. on £225 (= £22 10s.) and 20 per cent. on £390 (= £78), that is, £100 10s.

Quantitative deduction is the commonest type of deduction in the exact sciences, but it is common enough even elsewhere. It would not be very difficult to devise an abstract formula for it, but it is hardly worth while. For it really involves no new logical principle. It consists in bringing certain cases or classes of cases under a general law which embodies some quantitative formula. What distinguishes it from merely qualitative deduction is the supplementary calculations required in the way of proportioning the general formula to the precise quantities of the cases contemplated. The calculations are sometimes very complicated and difficult—and that may be one reason why some students prefer logic to mathematics.

CHAPTER XI

ABRIDGED SYLLOGISMS AND CHAINS OF SYLLOGISMS

§ 1. *The Order of Propositions in the Syllogism as a Common Form of Argument.*

Syllogisms are very common forms of argument or reasoning. Their frequency is obscured by certain facts which we have to consider next.

In the first place, as a matter of convenience in the exposition of the different types of syllogisms, they are usually arranged, in books on Logic, in a certain order—major premise first, minor premise next, and conclusion last. There is nothing sacred about this order, any other order would do equally well; only it saves time to have some uniform arrangement for purposes of exposition, because the character of the several constituent propositions can be recognized at a glance. In actual argument, the arrangement varies in all possible ways. Perhaps the commonest arrangement begins with the conclusion, so as to make clear at once what one is driving at. This is nearly always the case when there is a reasoned answer to a question. But whatever the sequence of propositions in actual argument, there should be no difficulty in distinguishing the premises from the conclusion, or the major premise from the minor premise, though even this requires some practice in the analysis of arguments.

Let us consider an illustration or two. Take the argument, "Triangles inscribed in a circle with the diameter for base and the vertex on the circumference have the sum of the squares on their two sides equal to the square on their base For such triangles are

right-angled at the vertex; and all right-angled triangles have the sum of the squares on the sides subtending the right angle equal to the square on the base (or hypotenuse)." Here the order of propositions is, conclusion, minor premise, major premise. If re-arranged, for purposes of easy scrutiny, in the conventional order adopted in this book, it would fall into the following scheme:

$$MaP$$
$$SaM$$
$$\therefore SaP.$$

Take another example. Suppose a teacher asks what part of speech the word *planet* is, and why? The pupil would probably say: "*Planet* is a common noun, because it is a word which can be applied in the same sense to any one of a certain class of things, and such words are common nouns." Here, likewise, the reasoned answer begins with the conclusion, goes on to the minor premise, and ends with the major premise. In logical form and character it is the same as the preceding syllogism.

§ 2. *The Abridgment of Syllogisms and the Universe of Discourse.*

In the second place, owing to the laudable tendency to be brief, and to put some trust in the intelligence of our fellows, syllogisms are usually abridged, in actual discourse, by the omission of one or other of the constituent propositions, though these are obviously implied, or taken for granted. To insist on each syllogism being expressed completely on every occasion would be as absurd as the pedantry of a

nursery governess who insists on complete sentences. An abridged syllogism is called an *enthymeme*, and is said to be of the first, second, or third order according as the omitted proposition is the major premise, the minor premise, or the conclusion.

In conversation, and in discussion generally, there is a wise tendency to be brief, by omitting whatever is obvious to an intelligent person. No doubt there are occasions when it is safest to be as explicit as possible. That is why, for instance, a lawyer's brief never is brief. But in ordinary intercourse there is as little inclination to regard legal documents as a model of self-expression as there is to regard legal process as a model of social etiquette. What happens for the most part is this. There is a mutual understanding about the general topic, or sphere of reference, which is under discussion. This sphere of reference is known as *the universe of discourse* or *limited universe* [1] (or *suppositio*, in Latin). And the fact that the conversation or discussion is understood (informally, of course) to be limited to a certain range of topics, instead of being directed to the world at large, enables the speakers to be less explicit, and consequently more brief, than would otherwise be the case. Thus, for example, if the general theme under discussion is modern literature and somebody remarks that "Machiavelli is rather long-winded," it will be understood that the reference is not to the historical person, but to the novel. In fact "Machiavelli" stands for "the novel Machiavelli," just as, when speaking of portraits, "Gladstone" would stand for "the portrait

[1] A full account of the conception of *universe of discourse* will be found in the author's *Studies in Logic* (Cambridge University Press), Chap. III.

of Gladstone," and so forth. Of course, there are people who insist on taking us too literally, just as there are others who will leave nothing to our imagination or to our understanding; we must accept them with resignation, like other trials, if we cannot escape them. Now sometimes whole arguments are abridged, thanks to mutual understanding, just as at other times single sentences or phrases are. In such cases the real argument as a whole cannot be duly evaluated unless all the omissions are made good, and taken into account. Sometimes an argument is really much stronger than appears at first sight when the omitted but assumed links have not yet been interpolated. But at other times an argument in its abridged state may appear much stronger than it is when completed, because some of the suppressed premises may appear more than doubtful when stated explicitly. In some cases the abridgment of an argument is due not so much to want of time as to lack of candour. Now it may or may not be frequently necessary to examine the validity of one's own or other people's arguments; but when it is necessary to do so, then the whole argument must be set out as explicitly as possible, by supplying all the assumed but suppressed propositions. When the argument consists of a single abridged syllogism this is not a difficult matter, and expertness with single syllogisms enables one to cope readily with chains of abridged syllogisms.

And now for a few simple illustrations of abridged syllogisms (or enthymemes). The examples given in § 1 may serve our purpose. In an actual geometry class, in which the Pythagorean theorem had just been dealt with, the argument about triangles inscribed in a circle with its diameter for base would ordinarily

be abridged by the omission of the last proposition, which is the major premise. If, on the other hand, what had most recently been explained was that such triangles are right-angled, the minor premise would probably be omitted as something obvious, while the major would be stated. Similarly with the grammar question. If the children had just been taught the definition of a common noun, the answer would probably omit the major premise; if not, it might omit the minor premise. Sometimes, again, the premises are given, but the conclusion is not stated. This is usually the case with suggestions, and with insinuations, especially with unpleasant insinuations. For example, "People who trade honestly don't get rich so quickly as Mr. X. did." This is equivalent to: *No people who work honestly get rich so quickly, Mr. X. is a person who did get rich so quickly, [Therefore Mr. X. is not honest].*[1]

$$MeP$$
$$SaM$$
$$[\therefore SeP].$$

Lastly, the following instance may show, in a simple way, how easy it is to mislead one by suppressing an inconclusive premise. If someone says "Miss Smith cannot yet be thirty years of age, because she has no Parliamentary vote," probably few people would suspect its accuracy; but if the major premise, perhaps obscurely in the speaker's mind, were stated explicitly, namely, "All women without a vote are under thirty," most people would recognize its inaccuracy. In the

[1] Brackets are used to indicate that the proposition enclosed in them was omitted from the wording of the original argument, but assumed by it.

example just given the conclusion followed from the premises, only the major premise was false—the form of the syllogism being

$$\begin{array}{l}[MaP] \\ SaM \\ \therefore SaP\end{array}$$

Sometimes, however, the assumed but suppressed premise is true, only it does not support the inference drawn. In the foregoing illustration, for instance, the premise in the speaker's mind may have been "All women voters are thirty years of age or over." In that case the form of the syllogism would be

$$\begin{array}{l}[MaP] \\ SeM \\ \therefore SeP\text{—an incorrect conclusion.}\end{array}$$

§ 3. *Chains of Syllogisms and of Abridged Syllogisms.*

In the third place, it is not very often that a problem can be settled with the aid of one syllogism, though such cases are by no means uncommon. For the most part arguments involve several interconnected syllogisms (or chains of syllogisms, or *polysyllogisms*, as they are called)—to say nothing of other forms of inference—and as each syllogism tends to be abridged, in the way just described, what we commonly find are chains of abridged syllogisms. Naturally, such arguments are not readily recognized, or easily deciphered, by those who have only a very superficial acquaintance with Logic, and have never made a serious attempt to study arguments in the flesh, so to say.

A *chain* of syllogisms, as distinguished from a mere group of disconnected syllogisms, is characterized by

the fact that each syllogism either supports, or is supported by, the other—there is real connection, or dependence, between them. A supporting syllogism is usually called, in relation to the supported or dependent syllogism, a *prosyllogism* ; the dependent syllogism, in relation to its *prosyllogism*, is called an *episyllogism*. The prefixes *pro-* and *epi-* mean, of course, *before* and *after*, respectively ; but in this case the reference is not to sequence in time, but to *logical* sequence—the conclusion of the prosyllogism is used as a premise of the episyllogism, which is therefore logically dependent on the former, whichever syllogism may be stated first. Take, for instance, the following argument : *Large-scale production means an increase in real income, for it cheapens articles of ordinary consumption, and so makes our incomes go farther than they would otherwise. Now an increase in real income is obviously a boon to people of small means. Therefore large-scale production is a boon to people of small means.* Here we have two syllogisms. In the first the order of statement is that of conclusion first, minor premise next, major premise last. In the second syllogism the major premise is stated first, and is followed at once by the conclusion—the minor premise is supplied by the conclusion of the first syllogism, and is therefore not repeated. Set out in the conventional order the argument has the following form :

$$\left.\begin{array}{r} MaP \\ SaM \\ \therefore SaP \end{array}\right\} \textit{Prosyllogism.}$$

$$\left.\begin{array}{r} PaQ \\ [SaP] \\ \therefore SaQ \end{array}\right\} \textit{Episyllogism.}$$

Now the actual order of statement might just as well have been reversed: *Large-scale production is a boon to people of small means, because it means an increase in their real incomes, which is a boon. For large-scale production cheapens articles of ordinary consumption, and so makes our incomes go farther than they would otherwise.* Symbolically it runs:

$$\left.\begin{array}{r} SaQ \\ \because\ SaP \\ PaQ \end{array}\right\} \textit{Episyllogism.}$$

$$\left.\begin{array}{r} [SaP] \\ \because\ SaM \\ MaP \end{array}\right\} \textit{Prosyllogism.}$$

Here, though the order of statement is reversed, the supporting syllogism being stated after the supported syllogism, yet logically the supporting syllogism remains the prior or *pro*-syllogism. If an argument is arranged in the first of these ways, so that it passes from pro-syllogism to episyllogism, that is, always towards an episyllogism, it is said to be episyllogistic, or progressive, or synthetic. If, on the other hand, it is expressed in the second way, passing from episyllogism to prosyllogism, that is, always towards a prosyllogism, it is said to be prosyllogistic, regressive, or analytic. A progressive chain of abridged syllogisms is called a Sorites; a regressive chain of abridged syllogism is called an Epicheirema. But usage is not uniform in these matters, and there is no particular virtue in these technical expressions. The main thing is to be able to recognize the real character of such chains of syllogisms, to analyse them into the constituent single syllogisms, and note their conformity or otherwise to the general rules of the syllogism.

§ 4 *Degrees of Complexity, or Linear and Systematic Inference.*

Chains of syllogisms and of abridged syllogisms may obviously be of all degrees of complexity. The above example was of the simplest possible type. Here is a symbolic illustration of a more complex type, which would not be difficult to match in *Euclid* or elsewhere.

$$
\begin{array}{llllll}
 & MaP & \because [NaP], & MaN & \because ZaN, & [MaZ] \\
 & SaM & \because RaM, & [SaR] & & \\
\therefore & SaP & . & & & \\
 & PaQ & \because [VaQ], & PaV & \because [TaV], & PaT \\
 & [SaP] & & & & \\
\therefore & SaQ & & & &
\end{array}
$$

In the simplest types of polysyllogisms, or sorites, the final inference is arrived at in a comparatively simple and straightforward manner, and the symbolic arrangement is in a straight vertical line; but as it gets more and more complicated, the argument, even when set out symbolically, assumes the appearance of a more and more complicated figure, or complex interconnected system. Hence we may distinguish between the comparatively simpler *linear* inference, and the more complex *systematic* inference. But it would be an obvious blunder to identify syllogistic inference with linear inference.

The degree of complexity of an argument varies partly with the extent of the thinker's knowledge, or beliefs, about the various topics relevant to the problem, and partly with the extent to which his views on these relevant topics are shared by those to whom the argument is addressed, if it is addressed to others. The larger the number of connected topics

on which the thinker already has definite views, the simpler will be the process of inference by which he endeavours to solve his immediate problem. Similarly the greater the agreement between the thinker and his audience on connected topics, the simpler will the argument be; and the greater the differences, the longer and the more comprehensive the discussion. That is why an argument which is sufficient for the followers of one party or school of thought rarely satisfies those of another. If the difference extends to fundamental principles, then the argument is apt to become metaphysical and—interminable.

CHAPTER XII

HYPOTHETICAL PROPOSITIONS AND INFERENCES

§ 1. *Categorical and Hypothetical Propositions.*

The objects of our thought vary enormously in respect of concreteness (or, its opposite, abstractness). Sometimes we think of and about some particular object or situation—some individual person, some domestic pet, some particular planet, and so on. Sometimes we think about types of objects, rather than individual members as such—human beings generally, horses or dogs generally, planets or stars in general. At other times the objects of our thought are even less concrete, or more abstract; we may think about general laws of human development, or about the general anatomical structure of quadrupeds, or the general conditions and laws of their evolution, or about the laws of planetary motion, the law of gravitation, and so on. The more general considerations are more abstract in the sense that they concern certain characteristics or properties and their mutual relations without special regard to their particular embodiments or settings.

The term *concrete* is derived from the Latin *concrescere* (to grow together), and anything which is thought of as a centre, or a coalescence, of many attributes and relations is said to be regarded as concrete, and the more concrete, the greater the number of attributes and relations associated with it. On the other hand, when the attributes or relations are thought of more or less apart from the things or situations which they characterize, they are said to be regarded abstractly.

There are various degrees of abstraction—a general term like *Englishman* is more abstract (or less concrete) than a singular term like *John Locke*, and a more general term like *man* is more abstract (or less concrete) than the term *Englishman*, while the name of a quality, like *colour*, or a relation, like *partnership*, is more abstract still. Nowadays only terms like the last two examples are usually called *abstract terms*, and general class names like *man* are called concrete; but not so long ago such general names were also called abstract names, because of the abstraction from the varying qualities, etc., of individuals and sub-classes which they involve.

Now propositions vary considerably in certain respects according as their terms are concrete or abstract. From the point of view of human knowledge it is obvious that the amount of knowledge embodied in a singular proposition (that is a proposition having a singular term for subject) cannot as a rule be compared in extent with that embodied in a general proposition; and, as will appear soon, the higher achievements of science are embodied in highly abstract propositions.

Again, one may assert something of a concrete subject without knowing in virtue of which attributes of the subject the predicate belongs to it. Such assertions are sometimes described as assertions of brute fact, in contrast with assertions of real connections between conditions and results. This distinction, and the other distinction just explained, are intimately connected. Assertions of brute fact are naturally expressed in the more concrete type of proposition; assertions of connection between conditions and results (in the widest sense of these terms) are as naturally expressed in the more abstract types of proposition.

Broadly speaking, the categorical type of proposition is the natural vehicle of the more concrete assertions of brute fact; the hypothetical proposition is the natural vehicle of the more abstract assertions of universal connections. It must not be forgotten, however, that in actual use the distinction between the two types of proposition is sometimes effaced, more or less abstract assertions of connection being expressed in the categorical form of proposition, while concrete assertions of brute fact, or of arbitrary stipulation, are expressed in the hypothetical form of proposition.

§ 2. *The Meaning and Implications of the Hypothetical Proposition.*

In the hypothetical form of proposition we have not merely a subject and a predicate, but an antecedent and a consequent, or a condition and result, between which a connection is asserted. Symbolically its general form is

If A, then C,

where *A* represents an antecedent, or condition, and *C* a consequent or result. For example, *If a triangle is equilateral, it is equiangular*; *If the temperature of a gas is raised, its volume is increased* (the pressure being constant); *If the supply of a commodity falls short of the demand, the price tends to rise* (in the absence of Government control). In all these cases there is an assertion of a general connection between an antecedent and a consequent, without special reference to actual or concrete cases. Of course, the assertion has application to concrete cases. In one way or another, assertions are usually grounded in reality, and have reference to the world of reality. Life is too short and strenuous

to be spent on unrealities. In one very real sense, the very abstractness of abstract assertions is only intended to give them a wider applicability, that is, to make them applicable to a wider range of facts. For that reason it is always possible to restate the more abstract assertions in more concrete form, to reduce the hypothetical form *If A, then C* to the categorical form *Every case of A is a case of C*. Thus the above examples of hypothetical propositions can be expressed as categorical propositions in the following way: *Equilateral triangles are equiangular; A gas, the temperature of which is raised, increases in volume; A commodity the supply of which falls short of the demand tends to rise in price.*

At the same time there are such things as suppositions which, though based on facts, may still be merely speculative, and may not refer to any actual instances. One might, for example, in the light of what is known about the effects of variations in temperature on all kinds of substances, think of what might happen to various substances if reduced to a temperature of absolute zero, although no actual instance of that temperature is known. Such suppositions, if they are to be expressed unambiguously, should be expressed only in the hypothetical form, and not in the categorical form, which would probably mislead people into supposing that the reference is to actual instances of the phenomenon in question.

The main feature, then, of the hypothetical type of proposition is the assertion of a connection between an antecedent (or a condition) and a consequent (or a result) Now the nature of the connection between the antecedent and the consequent must be grasped clearly, if misinterpretation and invalid inferences are

to be avoided. In all cases in which the form of assertion *If A, then C* is employed intelligently, it means that the antecedent (A) cannot be true without the consequent (C) being true likewise. We have just stated that, expressed categorically, it means AaC (*Every case of A is a case of C*); and to suppose that in some cases the antecedent might also be true although the consequent is not true, would be equivalent to asserting AoC, its contradictory. The consequent, then, must be admitted to be true, whenever the antecedent is true, if we commit ourselves to an hypothetical assertion; so that if in any case, or cases, the consequent is not true, then the antecedent cannot be true either. We can express this by saying that *If A, then C* implies *If not C, then not A*. On the other hand, though A cannot be true without C, yet C may be true without A. *If A, then C*, in other words, does *not* imply *If C, then A*. This is clear if we express them in categorical form. *If A, then C* means AaC, *If C, then A* means CaA. But we have seen that universal affirmative proposition may not be converted simply. AaC does *not* imply CaA, only CiA, that is, *Some cases in which the consequent is true are cases in which the antecedent is true*, which is not the equivalent of *If C, then A*. To treat *If A, then C* as though it implied *If C, then A* is essentially the same mistake as to convert SaP into PaS, though the fallacies have different names, namely, *the fallacy of consequent*, and *the fallacy of illicit conversion*, respectively. Since, then, the consequent (C) may be true even when the antecedent (A) is not, it follows that one may not infer the falsity of the consequent from the falsity of the antecedent—in other words *If A, then C* does *not* imply *If not A, then not C*. There are, indeed, some cases in which

If A, then C, and *If C, then A*, and *If not A, then not C* are all true. This happens whenever the antecedent and the consequent are *reciprocal*, that is, each implies the other, as, for instance, in the case of *If a triangle is equilateral it is equiangular*, where it is also true that *If a triangle is equiangular it is equilateral*. But such reciprocity is not usual, and must not be assumed without special evidence. Where the reciprocal relation does hold good one is, of course, entitled to express it in the two propositions *If A, then C* and *If C, then A*. In such cases the proposition *If C, then A* is not implied by *If A, then C* but is an independent assertion, and the proposition *If not A, then not C* is not implied by *If A, then C* but by *If C, then A*.

Briefly, then, the hypothetical form *If A, then C* implies (i) that in every case in which *A* is true *C* must be true, and (ii) that in every case in which *C* is not true *A* cannot be true (*If not C, then not A*). But it does *not* imply (iii) *If C, then A*, nor (iv) *If not A, then not C*.

It will have been noticed that the antecedent and the consequent of an hypothetical proposition consist each of a categorical proposition of the form *S is P* or *S is not P*. The antecedent and the consequent may be either affirmative or negative, and the symbols *A* and *C* must not be regarded as standing for affirmations only. Just as *x*, *y*, *z*, etc., may stand for negative as well as for positive quantities, just as *S*, *M*, *P*, etc., may stand for negative as well as for positive terms, so *A*, *C*, etc., may stand for a negative as well as for an affirmative antecedent, or consequent. Conversely, just as $-x$ may be a positive quantity, and $\bar{S}$ a positive term, so *not* $-A$ (or $\bar{A}$) and *not* $-C$ (or $\bar{C}$) may stand for an affirmative antecedent or consequent.

Usually an hypothetical proposition with a negative consequent is regarded as a negative proposition. It certainly can be so regarded and treated. But if the foregoing remarks are borne in mind the whole business of formal inference from hypothetical premises is considerably simplified. It becomes possible to treat all hypothetical propositions as affirmative, even if their antecedents or consequents happen to be negative. This is legitimate, because, after all, the main function of an hypothetical proposition is to *affirm* a connection between the antecedent and the consequent; and the connection is *affirmed* just as much when the antecedent and the consequent are (either or both) negative as when they are affirmative. Thus, for instance, the assertion *If a triangle is not equilateral it is not equiangular* is of the type *If A, then C* just as much as the assertion *If a triangle is equilateral it is equiangular*, though, naturally, if the former is symbolized as it is symbolized here, the latter, if it occurs in the same argument, will have to be symbolized *If* $\bar{A}$, *then* $\bar{C}$. The form remains the same, only the terms (the antecedents and consequents) are different. We can, accordingly, treat all hypothetical propositions as affirmative. The denial of an hypothetical takes the form of asserting another affirmative hypothetical proposition having the same antecedent but a contradictory consequent. The contrary will be universal; the contradictory will be either particular or modal.[1] Thus, for example, the contrary of *If A, then C* will be *If A, then* $\bar{C}$; its contradictory will be *If A, then sometimes* $\bar{C}$ or *Sometimes if A, then* $\bar{C}$, or *If A, then maybe* $\bar{C}$, or *If A, then not necessarily C.*

[1] Propositions of the form *S may be P*, *S need not be P*, *S must be P*, *S cannot be P*, are called modal.

This brings us to the question of *particular* hypothetical propositions. Strictly speaking hypothetical propositions, when properly used, should only be universal, not particular, because if there is a real connection between the antecedent and the consequent, the proposition is universal; and if there is no such general connection, the assertion should be expressed in the categorical form, not in the hypothetical form. It will be found that in a particular hypothetical proposition the particle "if" really means "when." The difference between *if* and *when* is this: *if* introduces a *condition* of a certain event, etc., and a real condition is universal, and is best expressed in an hypothetical proposition; *when* introduces *instances* of a certain event, etc., and an assertion about instances, as already explained, is best expressed in the categorical form of proposition. The fact remains, however, that people do sometimes use particular hypothetical propositions, which have to be dealt with accordingly.

§ 3. *Pure Hypothetical Syllogisms.*

In the light of the foregoing explanations there should be no real difficulty in dealing with the immediate inferences from hypothetical propositions, or with mediate inferences from two hypothetical premises. In either case the hypothetical expressions can, if necessary, be expressed in categorical form, and then treated in the way already explained in connection with categorical propositions and inferences. An example or two may be given here of syllogisms with two hypothetical premises, or pure hypothetical syllogisms, as they are called.

If the rays of light coming from the fixed stars are subject to gravitation they will be bent by planets near their path to the earth ;

If the rays of light, etc., are material they are subject to gravitation ;

∴ *If the rays of light, etc., are material they will be bent by planets near their path to the earth.*

The form of this syllogism obviously is :

$$\left.\begin{array}{r} \textit{If } B\textit{, then } C\text{,} \\ \textit{If } A\textit{, then } B\text{,} \\ \therefore \textit{If } A\textit{, then } C\text{,} \end{array}\right\} \begin{array}{c}\text{and}\\ \text{corresponds}\\ \text{to}\end{array} \left\{\begin{array}{r} MaP \\ SaM \\ \therefore SaP. \end{array}\right.$$

Here is an example with a negative consequent :

If a triangle is equiangular it is not right-angled ;

If a triangle is equilateral it is equiangular ;

∴ *If a triangle is equilateral it is not right-angled.*

This can be regarded as having the same form as the preceding example, but with a negative consequent in the major premise. Or it can be treated as having a negative major premise, in which case its form will be as follows :

$$\left.\begin{array}{r} \textit{If } B\textit{, then not } C\text{,} \\ \textit{If } A\textit{, then } B\text{,} \\ \therefore \textit{If } A\textit{, then not } C\text{,} \end{array}\right\} \begin{array}{c}\text{corresponding}\\ \text{to}\end{array} \left\{\begin{array}{r} MeP \\ SaM \\ \therefore SeP. \end{array}\right.$$

Of course, C and P in the two cases have contradictory meanings.

Finally, an example of an invalid syllogism :

If a man is guilty he is uncomfortable under cross-examination ;

If a man is nervous he is uncomfortable under cross-examination ;

∴ *If a man is nervous he is guilty.*

The form of this syllogism is :

$$\left.\begin{array}{l} \textit{If } C, \textit{ then } B, \\ \textit{If } A, \textit{ then } B, \\ \therefore \textit{If } A, \textit{ then } C, \end{array}\right\} \begin{array}{c} \text{and} \\ \text{corresponds} \\ \text{to} \end{array} \left\{\begin{array}{l} PaM \\ SaM \\ \therefore SaP. \end{array}\right.$$

This involves the fallacy of undistributed middle term.

§ 4. *Mixed Hypothetical Syllogisms.*

In addition to the mediate inferences which may be drawn from two hypothetical premises, it is also possible to draw mediate inferences from an hypothetical major premise and a categorical minor premise. Such mediate arguments are known as ***mixed hypothetical syllogisms***, or hypothetico-categorical syllogisms. After what has already been said above about the meaning and implications of the hypothetical type of proposition very little need be added to explain the mixed hypothetical syllogisms. With a major premise of the form *If A, then C* there are only two ways in which a categorical minor premise can mediate an inference. The categorical minor must either (i) posit the antecedent (*A*), or (ii) reject the consequent (*C*). In the former case (i), the consequent is accepted as conclusion ; in the latter case (ii), the conclusion denies the antecedent. So we obtain two main types of mixed hypothetical syllogism, which are known respectively as *Constructive* and ***Destructive***, and may be symbolized as follows :

Mixed Hypothetical Syllogisms

(i) *Constructive*	(ii) *Destructive*
If A, then C	*If A, then C*
A	*not C* (or $\bar{C}$)
$\therefore$ *C*	$\therefore$ *not A* (or $\bar{A}$)

For example:

(i) *If rays of light are material, they are subject to gravitation,*
Rays of light are material,
∴ *Rays of light are subject to gravitation.*

(ii) *If the rings of Saturn were non-material they would be invisible,*
The rings of Saturn are visible,
∴ *The rings of Saturn are material.*

It makes no fundamental difference to the form of the mixed hypothetical syllogism whether the antecedent and consequent are (both or either) affirmative or negative. But it should be remembered, of course, that the positing of a negative antecedent, or consequent, will give a negative minor premise, or conclusion, and the rejection of a negative consequent, or antecedent, will give an affirmative minor premise, or conclusion. So that the constructive or destructive character of a mixed hypothetical syllogism does *not* depend on the quality of the minor premise, or conclusion, but on their relation to the antecedent and the consequent of the major premise. A syllogism may be destructive even if its minor premise, or conclusion, is affirmative, and it may be constructive although the minor premise, or conclusion, is negative. The first of these points is illustrated by the preceding example (ii); the second point is illustrated by the following argument:

If carbon is not metallic it is not capable of powerful magnetic influence;
Carbon is not metallic;
∴ *Carbon is not capable of powerful magnetic influence.*

§ 5. *Abridged and Concatenated Hypothetical Syllogism.*

The remarks made in Chapter XI about abridged categorical syllogisms and chains of categorical syllogisms apply also, *mutatis mutandis*, to hypothetical syllogisms, pure and mixed. No special treatment is consequently necessary.

CHAPTER XIII

ALTERNATIVE (OR DISJUNCTIVE) PROPOSITIONS AND INFERENCES

§ 1. *The Meaning and Implications of the Alternative Proposition.*

Having considered categorical and hypothetical propositions, their meaning and their implications, we must consider next the disjunctive, or alternative, type of proposition. The essence of what is commonly called a disjunctive proposition is that it asserts that one or other of certain alternatives holds good. Its symbolic form may be best expressed thus: *Either* A_1, *or* A_2, where A_1 and A_2 stand for categorical propositions, such as *S is M*, *S is P*, *P is Q*, *S is not M*, etc. Just as the hypothetical proposition asserts a connection between *A* and *C*, and says in effect that the truth of *A* involves the truth of *C*, so the alternative proposition asserts that one of the alternatives (A_1 or A_2) is true, that both are not false.

To understand clearly the meaning and implication of the alternative type of proposition, it is necessary to realize how it comes to be used. Sometimes it arises out of the classification of things, qualities, etc., into classes and sub-classes. Lines, for example, are divided into the two sub-classes, right lines and curves. We consequently say *Lines are either straight or curved*, which means little more than *Some lines are straight, and some are curved*. Similarly, British subjects consist of two sub-classes, namely, British-born, and naturalized. So we say *British subjects are either British-born or naturalized*, which, again, means little more than that *Some British subjects are British-born*,

and some are naturalized. Of course there may be, and there frequently are, more than two sub-classes. For example, *Triangles are equilateral, or isosceles, or scalene,* which means little more than that *Some triangles are equilateral, some are isosceles, and some are scalene.* Now such propositions, though alternative in form, are really, as the second statement in each case shows, categorical propositions, or combinations of categorical propositions. But suppose now that we have to assert something about a subject, say S, which we know to be included in a certain class, say P, which has certain sub-classes, say p_1 and p_2. Now we may simply assert categorically *S is P*, which we could assert even if we did not know the sub-classes of P. But if we want to utilize our knowledge of the sub-classes of P we shall naturally assert *S is either* p_1 *or* p_2. For example, we may assert categorically that *Mr. X is a British subject*, or we may assert that *Mr. X is either a British-born or a naturalized British subject.* We know that one of the alternatives must be true, and if we should discover subsequently that, say, the first alternative is not true, then we shall know that the second is true. Now, if, as is usual, we call the wider class the *genus* of its sub-classes, and the sub-classes the *species* of their *genus*, then we can say that sometimes alternative propositions express our knowledge of the *generic* character of a subject, coupled with uncertainty about its *specific* character, though we know exhaustively what the various specific characters are. In such cases, namely, those based on our knowledge of the classification of the relevant objects, the alternatives are not only exhaustive, so that one of them must be true, but (since, in a sound classification, the sub-classes are always mutually

exclusive, as well as collectively exhaustive) they are also mutually exclusive, so that *only* one of them can be true. In other cases, however, as we shall see, the alternatives are not mutually exclusive, so that while one of them must be true, both may be.

There are cases, namely, in which the same kind of result can be achieved in various ways. The results achieved by such different means or methods are never precisely the same, but for some practical purposes they may be sufficiently similar to be regarded as essentially the same. Now suppose we know that there are only a certain number of ways, or means, say two (for the sake of simplicity), by which a certain kind of result can be obtained, then, although we cannot say categorically that it was, or will be, brought about in one way, we can still assert that one or other of these ways was, or will be, responsible for that result. For example, suppose we know that there are only two ways in which variations in the volume of a gas can be effected, namely, by changing its temperature, and by varying its pressure. In that case even if we do not know how exactly, in a particular instance, the change in the volume of a gas was effected, we can still assert, for instance, that *The increase in the volume of this gas is due either to an increase in temperature or to a decrease in pressure.* Similarly, if it is known that there are only two ways of achieving exceptional academic success, say, by exceptional cleverness and exceptional industry, then, in the absence of other information, it can be asserted of any relevant person that *He is either exceptionally clever or exceptionally industrious.* Now in these and similar cases the alternative proposition still asserts the truth of one of

the alternatives mentioned, but the alternatives are not mutually exclusive, so that although one alternative must be true, both may be true. In the above illustrations, for instance, the increase in the volume of the gas in question may be due to *both*, an increase in temperature and decrease in pressure; and the academic success of the person in question may be due to *both*, cleverness and industry. It will be seen, therefore, that it would be a mistake so to interpret the alternative form of proposition as to make the alternatives always mutually exclusive. The term *disjunctive*, which is commonly applied to alternative propositions, rather implies, or, at least, suggests strongly, that the alternatives are always mutually exclusive. It is, therefore, preferable to substitute the term *alternative*, which just brings out the fact that the propositions in question assert alternatives, without suggesting that they are mutually exclusive. In a great many cases the alternatives are as a matter of fact mutually exclusive. With a little care it is even possible so to express alternative propositions as to make the alternatives always mutually exclusive. For example, the proposition about the increase in the volume of the gas could be expressed in this way: *The increase in the volume of this gas is due to* (1) *an increase in temperature only, or* (2) *a decrease in pressure only, or* (3) *to both an increase in temperature and a decrease in pressure.* Or, more generally, the proposition *S is either P or Q* can be *restated* in the form *S is P only, or Q only, or both P and Q* (*S is* $P\bar{Q}$ *or* $\bar{P}Q$ *or* PQ), where the various possibilities are mutually exclusive. But the cumbrousness and pedantry of such restatement are rather against it, and, in any case, the fact remains that people do employ the

alternative form of proposition even when the alternatives are not mutually exclusive. The simplest way, accordingly, is not to regard the alternatives as mutually exclusive unless we have special reasons for it, that is, reasons other than the mere form of alternative assertion. We may conclude, therefore, that the alternative type of proposition asserts that one of the alternatives is true, and that it does *not*, as such, assert that *only* one of them can be true. From this it follows that the alternative proposition *Either* A_1 *or* A_2 implies *If not* A_1, *then* A_2 and *If not* A_2, *then* A_1; and that it does *not* imply *If* A_1, *then not* A_2 and *If* A_2, *then not* A_1. It is worth noting that in alternative propositions no real significance attaches to the order in which the alternatives are stated, so that, in the foregoing statement of what *Either* A_1 *or* A_2 implies and does not imply, the second proposition in each case was really unnecessary.

Alternative propositions are essentially affirmative. Whatever the alternatives may be, the proposition always asserts, in effect, that one of the alternatives *is* true. But the alternatives themselves (like the antecedents and consequents of hypothetical propositions) may be either affirmative or negative. Qualitatively, therefore, alternative propositions are always affirmative. The contradictory of *Either* A_1 *or* A_2 is *Neither* A_1 *nor* A_2. But this last proposition is not an alternative proposition, only a compound categorical proposition, meaning A_1 *is not true* (or *not* A_1) and A_2 *is not true* (or *not* A_2)—there are no *alternatives* asserted.

Quantitatively an alternative proposition should always be universal, never particular. The use of a particular alternative proposition, such as *Sometimes*

either A_1 *or* A_2, or *Some S's are either P or Q*, is simply a confession of ignorance of the other possibilities. The assertion in question would be expressed more accurately in two particular categorical propositions—*Some S's are P* and *Some S's are Q*. This should be obvious from such instances as *Some triangles are either equilateral or isosceles, Some heavenly bodies are either planets or comets.*

§ 2. *Pure Disjunctive Syllogisms.*

Having explained the meaning and immediate implications of the alternative type of proposition we may consider next the mediate inferences which may be drawn from two alternative propositions. The form of the mediating element (corresponding to the middle term of a pure categorical syllogism) is peculiar in this case. It should be obvious that two alternative premises, even with a common alternative, can yield no mediate conclusion. *Either* A_1 *or* A_2, and *Either* A_2 *or* A_3 can only be summarized in the proposition A_1 *or* A_2 *or* A_3. Similarly, *S is either P or Q* and *S is either Q or R* can only be summarized in the proposition *S is P or Q or R*. But then a bare summary is not a mediate inference. There is no term here merely mediating between others and forming no part of the result, as in the case of pure categorical and pure hypothetical syllogisms. The only way in which mediate inference is possible with two alternative premises is when an alternative of one premise contradicts an alternative of the other premise. Thus:

$$\left.\begin{array}{l} \textit{Either } A_1 \textit{ or } A_2 \\ \textit{Either not } A_2 \textit{ or } A_3 \\ \therefore \textit{ Either } A_1 \textit{ or } A_3 \end{array}\right\} \text{ or } \left\{\begin{array}{l} S \textit{ is either } P \textit{ or } Q \\ S \textit{ is either } \overline{Q} \textit{ or } R \\ \therefore S \textit{ is either } P \textit{ or } R \end{array}\right.$$

The following argument may serve as an illustration of this very rare type of argument. *A commodity is either produced on a large scale, or it is costly; And it is either in great demand, or it is not produced on a large scale; Therefore a commodity is either in great demand, or it is costly.*

This is the only type of pure alternative syllogism, abstracting from such minor variations as arise from differences in the quality of the alternatives, which (as already explained) may be either affirmative or negative.

The pure disjunctive syllogism is not very much like a syllogism, and is an uncommon type of argument, but it is quite valid when each premise exhausts the possible alternatives. Its formal validity may be shown as follows. The premise *S is either P or Q* implies that *if S is $\overline{Q}$ then it is P*; and the premise *S is either $\overline{Q}$ or R* implies that *if S is Q, then it is R*. From the two premises it follows that *whenever S is $\overline{Q}$ it is P* and *whenever S is Q it is R*. The cases in which S is R therefore include all the cases in which S is Q; and the cases in which S is P include all the cases in which S is $\overline{Q}$. Consequently, the conclusion *S is either P or R* exhausts all possible cases.

§ 3. *Mixed Disjunctive Syllogisms.*

It is possible to draw a mediate inference from an alternative major premise and a categorical minor premise. After what has already been said about the meaning and implications of the alternative type of proposition, it should be obvious that the only kind of case in which a mediate inference can be drawn from an alternative major premise and a categorical

minor premise is when the minor premise denies an alternative of the major premise. Thus:

$$\left.\begin{array}{l} \textit{Either } A_1 \textit{ or } A_2 \\ \quad \textit{Not } A_1 \\ \quad \therefore A_2 \end{array}\right\} \text{ or } \left\{\begin{array}{l} \quad S \textit{ is either } P \textit{ or } Q \\ \quad S \textit{ is not } Q \\ \therefore S \textit{ is } P \end{array}\right.$$

This is the only type of mixed disjunctive syllogism, allowing for the fact that there is no significance in the order of the alternatives of the major premise, and that the alternatives may be negative as well as affirmative, and that consequently the minor premise and the conclusion may be affirmative as well as negative. The following arguments may serve as illustrations of the mixed disjunctive syllogism.

(1) *The light by which we see the moon, when it is beyond the reach of the direct rays of the sun, is due either to the moon's own light or to earth-light* (that is, light reflected from the earth); *But it is not the moon's own light* (or the moon is not self-luminous);

∴ *It is due to earth-light.*

(2) *Heat is either a kind of substance* (stuff) *or a kind of energy ;*

But it is not a substance ;

∴ *It is a kind of energy.*

CHAPTER XIV

DILEMMAS

§ 1. *Principal Types of Dilemma.*

A *dilemma* is an hypothetico-disjunctive syllogism, or a mediate argument based on an hypothetical major premise and an alternative minor premise.[1] Now a little reflection will show that with a single hypothetical major premise it is impossible to employ a disjunctive minor premise, because a single hypothetical proposition presents no opportunity for an alternative assertion. With a single hypothetical proposition as major premise, the minor premise, whether it posits the antecedent or denies the consequent, is categorical, not alternative, and the whole argument is consequently a mixed hypothetical syllogism, not a dilemma. It is only when the major premise consists of two hypothetical propositions that the minor premise can be alternative, and either affirm alternatively the two antecedents, or deny alternatively the two consequents. The dilemma, like the mixed hypothetical syllogism, has two principal forms, the *constructive*, and the *destructive*. If the disjunctive minor premise posits (alternatively) the two antecedents of the hypothetical major premise, then the dilemma is called constructive; if the minor premise denies (alternatively) the two consequents of the hypothetical major premise, then the dilemma is called destructive. The following are the principal symbolic forms of the dilemma:

[1] Hence the dilemma is sometimes included among both mixed hypothetical and mixed disjunctive syllogisms. But to avoid ambiguity it is best to confine the terms *mixed hypothetical* and *mixed disjunctive syllogisms* to those cases in which the minor premise is a categorical proposition.

(1) *Complex Constructive Dilemma*

If A_1, *then* C_1, *and if* A_2, *then* C_2
Either A_1 *or* A_2
$\therefore$ *Either* C_1 *or* C_2

(2) *Complex Destructive Dilemma*

If A_1, *then* C_1, *and if* A_2, *then* C_2
Either not C_1 *or not* C_2
$\therefore$ *Either not* A_1 *or not* A_2

The following arguments may serve as illustrations of the two types of dilemma respectively:

(1) *If heat is a stuff its quantity must vary with the volume of the substance which contains it, and if heat is a kind of energy then its quantity must vary with the amount of energy expended;*
Heat is either a kind of stuff or a kind of energy;
$\therefore$ *The quantity of heat must vary either with the volume of the containing substance, or with the amount of energy expended.*

(2) *If an examiner is tender-hearted he will pass weak candidates, and if he is just he will reject them;*
But he must either reject them or pass them;
Therefore an examiner is either not tender-hearted or not just.

It is usual also to distinguish between *simple* dilemmas and *complex* dilemmas. The foregoing schemas and arguments are illustrations of complex dilemmas, because the major premise in each case has two distinct antecedents and two distinct consequents. When the major premise has the same consequent for both antecedents, or the same antecedent for both consequents, then the dilemma is called simple. If there are two antecedents and only one

consequent the simple dilemma can be constructive, but not destructive, because there are not two consequents to be denied alternatively in the minor premise, and the resulting argument, if destructive, cannot be a dilemma, though it can be a valid mixed hypothetical syllogism. When there are two consequents and only one antecedent in the major premise the resulting simple dilemma can be destructive, but not constructive, because there are not two antecedents to be posited alternatively in the minor premise, and so the resulting argument, if constructive, cannot be a dilemma, though it may be a valid mixed hypothetical syllogism. The principal forms of the simple dilemma are the following:

(3) *Simple Constructive Dilemma*

If A_1 *or* A_2, *then* C
Either A_1 *or* A_2
$\therefore$ *C*.

(4) *Simple Destructive Dilemma*

If A, then both C_1 *and* C_2
Either not C_1 *or not* C_2
$\therefore$ *Not A*.

The following arguments may serve by way of illustrations of the two types of simple dilemma:

(3) *If the miners have to work longer or to earn less, they will be dissatisfied;*
But they must accept either longer hours or reduced wages;
Therefore the miners will be dissatisfied.

(4) *If the coal industry were in a sound condition the miners and the mine-owners would be contented;*
But either the miners or the mine-owners are discontented;
Therefore the coal industry is not in a sound condition.

§ 2. *Difficulties and Faults of Dilemmas.*

The dilemma is a difficult form of argument merely because it happens so rarely that a problem, or situation, can be expressed exhaustively, or even adequately, in two alternatives; and when the alternatives are not exhaustive, then the conclusion is not valid. At the same time it must not be supposed that there is anything inherently wrong with the dilemma. The way in which dilemmas are sometimes used, or rather abused, for rhetorical purposes, or in jest, is rather apt to convey the impression that the dilemma is a kind of sophistical trick rather than a sound form of argument. But such a view is quite erroneous. When the alternatives expressed in the minor premise of the dilemma are exhaustive, and the terms employed are not ambiguous and misleading, the dilemma is quite sound. Nearly always, if not absolutely always, when a dilemma is invalid it is due to the fact that the minor premise does not exhaust all the possibilities, so that there remain possibilities leading to other results than those stated in the conclusion. And the defect is often concealed by ambiguous language which gives the minor premise the appearance of exhausting the alternatives when it really does not. Take, for example, the kind of arguments one sometimes finds in reports on inquests in connection with so-called peculiar people. The excuse for not calling in medical help usually runs somewhat as follows:

If the deceased was destined to recover, then medical aid was unnecessary, and if he was not destined to recover, then medical aid was futile;

But he was either destined to recover or not destined to recover;

Therefore medical aid was either unnecessary or it was futile. (In other words, *there was no point in getting medical help.*)

Here the dilemma looks sound, and yet most people feel that it is not. It looks sound because the alternatives stated in the minor premise appear to be exhaustive, for they look like a pair of contradictory alternatives (*destined* . . . and *not destined* . . .), and contradictory terms are collectively exhaustive. As a matter of fact, however, the alternatives are not exhaustive, indeed the most important possibility is omitted altogether, and its omission is veiled by a mere ambiguity. The ambiguity is this. In the major premise the second hypothetical proposition is only plausible because we assume that "not destined to recover" means "destined not to recover"; in the minor premise "not destined to recover" only looks like the contradictory of "destined to recover" (and therefore as its completing or complementary alternative) because it is taken in a much wider sense than "destined not to recover," as including, in fact, not only this alternative, but also the case of there being no destiny at all. Now, obviously, if there is no such thing as fatalistic destiny, then everything might depend on the medical help called in in case of illness. This is the common-sense view, yet this possibility is entirely omitted from the argument—to say nothing of the possibility of a conditional destiny depending on certain measures being taken. The real possibilities may be indicated in the following scheme. A man's lot may be either (*a*) *destined* or (*b*) *not destined*; and if (*a*) *destined*, then the destiny may be either (*i*) con-

ditional on certain steps being taken, or (*ii*) unconditional. The above dilemma as a matter of fact is tacitly based on (*a*) (*ii*) alone, and quietly ignores (*a*)(*i*) and (*b*) altogether.

§ 3. *The So-called Rebuttal of False Dilemmas.*

The commonest type of dilemma is the complex constructive, and as dilemmas are frequently invalid there has come into vogue a special device for refuting the complex constructive dilemma when it is suspected of inaccuracy. The device is known as that of *rebutting* a dilemma, and consists in transposing the consequents of the major premise, changing their quality, and then proceeding as usual. Let the original dilemma have the form:

If A_1, then C_1, and if A_2, then C_2
Either A_1 or A_2
$\therefore$ Either C_1 or C_2

Then the rebuttal will assume the following form:

If A_1, then not C_2, and if A_2, then not C_1
Either A_1 or A_2
$\therefore$ Either not C_2 or not C_1

For example, let the original dilemma be the one given on page 141 f, then its rebuttal will read as follows:

If the deceased was destined to recover, medical aid was not futile, and if he was not destined to recover, medical aid was not unnecessary;
But he was either destined to recover or not destined to recover;

Therefore medical aid was either not futile or not unnecessary. (In other words, *there would have been no harm in calling in medical aid.*)

For some reason or other the process of rebuttal has almost escaped criticism. It is possible that with an uncritical audience a so-called rebuttal may produce a favourable impression. As a rhetorical trick the device may, therefore, have some value. But logically it is worthless. Any complex constructive dilemma, even the soundest, can be "rebutted," and this alone should have made logicians suspicious of its value. The fact is that the "rebuttal" does *not* rebut the original conclusion at all. This should be obvious from a comparison of the original conclusion with the alleged refutation. The original conclusion is *Either* C_1 *or* C_2, and the alleged refutation is *Either not* C_2 *or not* C_1; but they are perfectly consistent with each other.

As has already been explained, the usual fault of an unsatisfactory dilemma is that the alternatives stated in the minor premise are not exhaustive. The most effective way of really rebutting a dilemma is to point out what possibilities have been overlooked, and to show up such ambiguities as may lurk in the argument.

§ 4. *Abridged and Concatenated Disjunctive Syllogisms.*

It only remains to point out that the account given in Chapter XI of abridged categorical syllogisms and chains of categorical syllogisms applies, *mutatis mutandis*, also to disjunctive syllogisms, pure and mixed, and to dilemmas.

INDUCTIVE LOGIC

CHAPTER XV

INDUCTIVE INFERENCE AND ASSOCIATED COGNITIVE ACTIVITIES

§ 1. *Inductive Inference.*

The inferences discussed in the preceding chapters were all such as could be drawn from given premises, or could be evaluated in the light of given premises, provided one had sufficient knowledge of the language in which they are expressed to understand the meaning and implication of the main types of propositions, singly or in certain combinations. This does not mean that all these discussions turned on purely verbal matters. Far from it, for propositions express judgments, or thoughts, and thoughts are concerned with reality. But, for reasons already explained, we assumed that the propositions constituting the premises were there somehow, and that we were only concerned with the unfolding of their implications, not with the problem of their origin or derivation. If, however, it be asked now how the requisite propositions are obtained, then the answer is that some are obtained by direct observation, some by intuition, some by inference from other propositions, and some by inference from facts of observation. Singular propositions and particular propositions are commonly the result of direct observation, or of inference from other propositions so obtained. But general propositions, though many of them are inferences from other general proposi-

tions, are in the last resort obtained as inferences, or generalizations, from observations, and are not the mere equivalents of observations, in the way in which many singular and particular propositions may be said to be. Moreover, there are even singular and particular propositions which are likewise not the bare equivalents of observations nor inferences from other propositions, but inferences from observations.

Now the methods by which general propositions, or generalizations, and certain particular propositions like those expressing certain statistical regularities, are obtained from observed data are known as the methods of science, and constitute the main problem of what is sometimes called *Methodology*, sometimes *Scientific Method*, and sometimes *Inductive Logic*. One or two points may be noted at the outset.

The methods of science can, of course, be described in general terms, and, when so described, can be shown to involve certain of the types of inference described in the preceding chapters. But the successful applications of the scientific methods involves a great deal more than that. It involves a familiarity with the facts investigated, and an insight into them that cannot be usefully formulated at all, but which may prompt conclusions which are felt to be sound in spite of certain shortcomings when gauged in the light of the rules of purely formal inference.

Again, when such generalizations have been made and formulated in clear propositions, these proposi-

tions can be made the starting-point of immediate and mediate inferences which may help one to explain present and past events, or to anticipate future events. But the transition from general proposition to events, though by no means so easy as to be fool-proof, is much easier than the transition from observed events to trustworthy generalizations. For one medical man who makes a new discovery there are probably more than ten thousand who do not, but who can safely and usefully apply old generalizations to new cases.

Again, a knowledge of the kind of inferences dealt with in the preceding chapters should be helpful in connection with the application of scientific methods. For, after all, what is aimed at in scientific investigation may be described, from one point of view, as the discovery of propositions from which the observed facts might have been inferred—the inferability of the facts from such propositions constituting usually the explanation of the facts. That is why *induction* is commonly described as the *reverse of deduction*. But the main point is to realize that the study of scientific method is inevitably less abstract than the study of inferences from propositions, and is consequently on a rather different footing.

For the better understanding of the problems of inductive logic it is advisable to have some insight into the chief mental activities by which knowledge, including science, is built up. We propose, accordingly, to take up the consideration, however brief, of these topics.

§ 2. *Observation and Experiment.*

Science is the creation of man. Nature, with all her regularities and irregularities, might have been just as real even if there were no men to observe and to study her. But there could have been no *science* without human beings, or beings like them. It is the spirit of man brooding over the stream of natural events that has given birth to science. For science is knowledge, and knowledge is the result of mental activities operating upon a world of objects. Now, speaking generally and without any attempt at psychological analysis at this stage, the mental activities which lead to scientific knowledge are roughly of two principal kinds, namely, processes of *Observation*, and processes of *Inference*. By Observation is meant the act of apprehending things and events, their attributes and their concrete relationships, also the direct awareness of our own mental experiences. By Inference, as already explained, is meant the formation of judgments on the strength of, or as a consequence of, other judgments already formed, it may be, on the ground of observations, or only entertained provisionally either for further consideration, or for the sake of argument. The broad distinction between observation and inference is sufficiently clear. But it is not always easy to draw the line between them, as will be shown, to some extent, later. And, unfortunately for clear thinking, people do not always realize that they are drawing inferences when they pass from particular observations to generalizations or to forecasts.

In the case of observation in the interests of science we may distinguish two principal kinds, namely : (*a*) *bare observation* of phenomena under circumstances which are beyond control, and (*b*) *experiment*, that is observation of phenomena under conditions which the investigator can control. Bare observation, as also experiment, may be assisted greatly by the use of scientific instruments, such as telescopes, microscopes, etc., also by selecting specially suitable times and places for making the observations ; but no scientific instruments, and no amount of trouble taken over the observation of the phenomena investigated, can be said to render the observation experimental in character unless the phenomenon observed and the circumstances of its occurrence are actually affected and controlled thereby. The chief advantage of experiment over bare observation is, that under experimental conditions it is usually easier to analyse accurately a complex phenomenon into its components, and to vary the circumstances of its occurrence in such a way that it is possible to draw reliable inductive conclusions concerning the connection between certain antecedents and consequents, or conditions and results. When phenomena and the circumstances of their occurrence are entirely beyond the control of the investigator, he is apt to overlook some important factors altogether, and to misjudge the function of others.

Inference, likewise, has two chief types, namely, *Induction* and *Deduction*. Inductive inference is the process of ascertaining some kind of order (class-

character, law, or system) among the phenomena observed and studied. Deductive inference is the process of applying either inductive conclusions or hypothetical concepts, laws, or regularities to suitable cases or classes of such cases. In the scientific study of natural phenomena, inductive inference plays the most important rôle, though deductive reasoning also contributes its share.

Observation and inference, however, are very complex processes, and it is advisable to consider some of the constituent processes which are of the utmost importance for all scientific investigation, but cannot be described as specific methods of science because they are really constituents of all, or nearly all, scientific methods properly so called. The cognitive processes referred to are those of analysis and synthesis, imagination, supposition and idealization, comparison, and the perception of analogies A brief account of these various processes follows, but no significance should be attached to the order of their exposition in the following sections.

§ 3. *Analysis and Synthesis.*

The discovery of order in the phenomena of nature, notwithstanding their complexity and apparent confusion, is rendered possible by the processes of analysis and synthesis, which are the foundation of all scientific methods. The objects and events which we observe are nearly always complex, but, mentally at least, we can always analyse them into their constituents or components. This process is helped by the comparison of two or

more objects or events which are similar in some respects, and different in others. But in its turn analysis facilitates more exact comparison. Having analysed the complex whole into its parts or aspects, we may tentatively connect one attribute of a thing with another, or one aspect of a thing with another, in order to discover a law; or we may, in imagination, synthesize again some of the attributes or aspects, and so form an idea of what is common to many objects or events. The process may be extended to classes of classes, whether of things or of events

The elements obtained by the analysis of different objects or events may also be synthesized in such a way as to form combinations the like of which have never been observed at all. In this way we form or acquire general and abstract ideas without which all higher knowledge, including science, would be impossible. In some cases, as in physics, chemistry, etc., the processes of analysis and synthesis can also be carried out materially (that is, objects can actually be broken up into their parts, or *vice versa*), and then scientific discovery may be greatly aided by the experimental variation of the conditions of the phenomena, in accordance with the direct methods of induction.

§ 4. *Imagination, Supposition, and Idealization.*

The presence of order in nature is not very obvious. The impression made by the observation of natural phenomena is, for the most part, one of bewildering confusion. How do we come to look for order at all? Why do we take the trouble to

discover classes, connections, or laws? The answer is fairly clear if we bear in mind the original character of human knowledge, including science. Knowledge was born in the service of life; it was, and in many ways still is, essentially an instrument of life; and life needs some kind of order in its environment, if it is not to be a perpetual groping in the dark.

Where possible life actually creates order of some degree, as may be seen in the habits of animals, in the customs, laws, and conventions of human society. In the case of natural phenomena, of course, man cannot *create* order, he can only look for it, try to discover it, if it is there. But it is the need for order, as an aid to life, that prompts man to search for it everywhere. If he succeeds, well and good; if he fails to discover order, or some special form of order, in any realm of facts, then he modifies his expectation, or maybe abandons it altogether, and turns his attention to other facts in the hope of discovering order there. Sometimes, indeed, the need for perfect order has been felt so keenly, in the face of the obdurate disorder of an imperfect world, that men like Plato and other idealistic philosophers have conceived an ideal, transcendental world over and above this world of ours. Here is shown, in an extreme form, the felt need for order which, in varying degrees, all intelligent human beings experience. And it is this felt need that always prompted mankind, and still prompts us, to try and discover order in Nature.

The actual search for order in nature follows more or less the usual modes of human conduct. It

begins with what is known as the method of trial and error, and, in the course of time, is characterized more and more by that insight and guidance which are the fruits of accumulated knowledge and experience. That is to say, at first, any kind of classification might be tried, based on any kind of resemblances, and any facts or events may be believed to be connected if they are observed to be conjoined. But, in due course, the mistakes are corrected in the light of subsequent experience, and the process of discovering order is carried out with far greater caution, with the help of, and with increasing regard for, the knowledge already acquired. Such a cautious, tentative attempt to discover order in any group of facts, by trying to fit a supposition that would make them appear orderly, constitutes what is known as the method of hypothesis.

An hypothesis is any tentative supposition by the aid of which we endeavour to explain facts by discovering their orderliness. All the methods of science may be said to depend on fruitful hypotheses. So long as it can be put to the test, any hypothesis is better than none. Without the guidance of hypotheses we should not know what to observe, what to look for, or what experiments to make, in order to discover order in nature. For observation not guided by ideas, even hypothetical ideas, is blind, just as ideas not tested by observation are empty.

Hypotheses or suppositions are, of course, used in everyday life, and in philosophy and in theology, as well as in science. In science, however, no

hypothesis is seriously entertained unless it can b put to the test of observation, either directly o. indirectly. Hypotheses may, of course, be true even if they cannot be tested or verified. On the other hand, hypotheses that can be tested by observation frequently turn out to be false, when so tested. Nevertheless, science has no use for *barren* hypotheses, that is, hypotheses which cannot be put to the test. Many hypotheses, which subsequently turned out to be false, were fruitful all the same, because they suggested lines of investigation which, though they led to the repudiation of those hypotheses, also led to the discovery of truths. But hypotheses which are barren at one time may become fruitful at a subsequent period, when suitable scientific instruments and processes have been invented. Thus, for example, most of the hypotheses relating to air were barren until Guericke invented the air-pump, and the chemists of the eighteenth century invented suitable processes for the analysis of air.

Intimately connected with the processes of imagination and supposition is the process of idealization, whose function in science has not hitherto met with due recognition. By idealization in science I mean the process of conceiving the ideal limit of some phenomenon that has been observed in various forms more or less approximating that limit, but never reaching it. The conception of ceaseless movement, implied in the first law of motion, is a case in point. The numerous uses of limiting cases in mathematics, and the conception

of a purely "economic man" in economics, are other instances. When supplemented by the process of hypostasization, that is, the process of treating an idea or a concept as though it were an existing object, the process of idealization may go a long way to explain the ideal constructions of mathematics, without our having to resort to a Platonic idealism or a Scholastic realism.

§ 5. *Comparison and Analogy.*

The observation of similarities and differences, aided by the processes of analysis and synthesis, constitutes one of the first steps in all knowledge, and accompanies its progress throughout. But there are degrees of similarity. Things, attributes, or events may be so similar that we regard them as being of the same kind, or as belonging to precisely the same class or type. On the other hand, there is a similarity which stops short of such close class-resemblance, and then we refer to it as analogy. In its wide sense the term analogy is applied to similarity of function, similarity of relationship, in fact, almost to any similarity short of that which characterizes members of the same class of things or events. Now, analogy also plays an important rôle in the advance of science. The acquisition or discovery of new knowledge is rendered possible by utilizing the knowledge already acquired. It is a process of apperceiving new or strange phenomena in the light of what is already known of other similar or analogous phenomena. In our search for order in any group of phenomena we naturally

attempt to "try on" any kind of order with which we are already familiar. Hence, analogy is a very fruitful guide to the formation of hypotheses or tentative orders of phenomena. Sometimes, indeed, what at first appeared to be a somewhat remote similarity or analogy may, on further investigation, turn out to be so close that what at first appeared new and strange is included in the same class as the old, by the aid of which it was apperceived. Thus, for example, lightning turned out to be the same kind of thing as an electric flash, and the movement of the moon was shown to be the same kind of phenomenon as the fall of an apple. This result, it is true, is not very common; but even in other cases analogy is very helpful. One need only think of the most important discoveries in the history of science, in order to realize the enormous value of analogy. Our conception of the solar system (the helio-centric theory) owes a great deal to the analogy of the miniature system of Jupiter and the Medicean satellites. Some of the most important discoveries in modern mathematics are due to the analogy, discovered by Descartes, between algebra and geometry. The wave-theory of sound was suggested by the observation of water-waves; and the undulatory theory of light was suggested by the analogous air-waves which transmit sound. The theory of natural selection by the struggle for existence was suggested to Darwin by his knowledge of the artificial selection by which breeders have produced the many varieties of domestic animals. And so forth.

It is important, however, to bear in mind also that analogy, as suggested, is not an independent scientific method but only an aid to the formation of hypotheses. Its sole service consists in originating hypotheses, and so suggesting lines of research in which scientific methods may be employed. By itself, analogy establishes nothing, notwithstanding the frequent reference one meets with to what is called "proof by analogy." The reason may be briefly indicated as follows. Generally speaking, what happens in so-called proof by analogy is this: some phenomenon or a class of phenomena, say *S*, is observed to resemble some other phenomenon or class of phenomena, say *Z*, in respect of some feature, say *M*; from this similarity it is concluded that *S* resembles *Z* also in respect of some other feature, say *P*, which *Z* is known to possess, but which has not yet been observed in *S*. Now, such a conclusion can only be justified if it can be shown that, directly or indirectly, *M* and *P* are connected by some law, for unless indeed there is some ground for supposing that *M* and *P* are connected in some way, the similarity between *S* and *Z* in respect of *M* is really irrelevant in considering their possible similarity in respect of *P*. But the question of the connection between *M* and *P* can only be decided by the inductive methods, not by the mere analogy. For example, the undulatory character of the transmission of light and sound, as already remarked, was suggested by the wave-motion of water; but only *suggested*. The hypothesis had to be verified by observation and experiment If

analogy alone were sufficient to warrant a conclusion it might have been assumed that, since the transmission of sound is analogous to that of light, the phenomena of polarization, which are found in the case of light, would also be met with in the case of sound. But analogy could only *suggest* this as an hypothesis, which subsequent inductive investigation has *not* verified. So that analogy, like all perception of similarity, and like analysis and synthesis, and imagination and supposition, must be regarded as an auxiliary or a preliminary to the inductive methods properly so called, rather than as an independent scientific method.

CHAPTER XVI

CIRCUMSTANTIAL EVIDENCE

§ 1. *General Character of Circumstantial Evidence.*

There is one kind of inference which, although it is not always concerned with generalization, is like induction inasmuch as it is inference from facts rather than from propositions, is a reverse process, and proceeds by way of hypothesis. We refer to inference from circumstantial evidence. It is a very common type of reasoning in the study of history and literature, and especially in daily life. But it is also employed in the service of science, although this is apt to be overlooked on account of the fact that science is mainly interested in the generalizations which usually *follow* the argument from circumstantial evidence when employed in science. We shall return to this point later on. First of all, however, it is necessary to explain and illustrate the general character of inference from circumstantial evidence.

Circumstantial evidence is particularly common in connection with attempts to trace criminals, but of course it is not confined to such cases. What is meant by *circumstantial evidence*? The term is commonly used by way of contrast with what may be described as evidence bearing directly on the main problem, and denotes evidence concerning, or consisting of, certain facts or factors which, so to say, *surround* the main event. For example, take the case of a theft. If the thief is caught in the

act, that is direct evidence of the most conclusive kind. Usually, however, people who plan a crime will take every precaution they can think of against red-handed discovery, in fact, against any kind of direct observation of their crime. Such direct evidence is, accordingly, uncommon. Frequently, however, there are " circumstances," that is occurrences surrounding the principal act, and more or less connected with it, which betray the criminal. It is these " circumstances " that constitute the *circumstantial evidence.* What usually happens is this. Certain objects or occurrences, observed in the neighbourhood of the locality where the crime was perpetrated, are felt to be connected with the crime or the criminal. These are traced as far as possible, and linked up with other facts and occurrences until they suggest an hypothesis about the identity of the criminal. The hypothesis, if satisfactory, will be such that it links up the otherwise disconnected objects and occurrences into one connected whole or system. And, in the absence of serious counter-evidence, the hypothesis which gives the most consistent and complete picture of the principal event in question and its circumstances will be accepted as the true account. It is important, of course, that no circumstances unfavourable to the hypothesis should be ignored. An arbitrary selection of circumstances may send innocent people to prison or even to the gallows. Such things have happened.

Perhaps the simplest illustration of inference from circumstantial evidence is the process of piecing together the parts of a picture puzzle. Here we have

a number of separate parts which are supposed to be connected somehow, or to belong together, and the solution of the puzzle takes the form of putting them together in such a way that they present a consistent and harmonious whole. Of course the problems of practical life are not often solved so completely as picture puzzles, or cross-word problems, which leave no parts over that cannot be accounted for.

Inference from circumstantial evidence is a very common method of historical reconstruction, even apart from the unravelling of political crimes. The task of the political historian may be described as that of so utilizing the available evidence as to reconstruct by means of it the drama of the events in question. It is in some ways like piecing together the pieces of a picture puzzle, but of a picture puzzle of which many parts are missing, while others have been damaged, or distorted, or faked. The result is that there is far more scope for the play of individual fancy in the case of historical reconstruction than there is in the assembling of the parts of an ordinary picture puzzle. What has been said of political history holds good likewise of literary history, and of all kinds of historical and biographical researches.

Inference from circumstantial evidence can be abundantly illustrated from the files of newspapers, and from political and literary histories. Any good account of, say, the chronology, etc., of the plays of Shakespeare (to say nothing of the Bacon-Shakespeare controversy) will afford examples of this kind

of inference. But the following quaint example, culled from the recent speculations of a contemporary archæologist, may serve our purpose.

Everybody is familiar with the expression "the island of the dead." Is the expression merely a poetic fancy, or does it express an early belief in the actual existence of such an island; and, if so, which island did it refer to? If we could cross-examine the early writers who used the expression they might tell us what they really believed. That would have been *direct* evidence. But that is not available. One has, consequently, to rely on "circumstantial evidence," that is, such allusions to "the island of the dead" as can still be read in old writings.

Now the principal relevant "circumstances" may be summarized as follows. According to Greek mythology the underworld of the dead is a land of darkness, where the sun never breaks through the mists and clouds. According to Homer, the land of the Cimmerii formed the entrance to the underworld, and this land was situated "at the end of the deep Ocean." The Styx, which the dead had to cross in boats, is also described as "an arm of the Ocean." And in the passage in the Odyssey, which describes the journey of the slain wooers across the Styx to the underworld, there is a reference to some "white cliffs" which they passed on their way. Now Pluto was not only the god of the underworld, but also of metal ores. The Cornish coast of Britain was well known to Phœnician mariners, who came there for tin. This coast also marked the limit of maritime adventures in those days ("the end of the

deep Ocean "). To the sunburnt Mediterranean the clouds and mists and pale complexions he encountered in Britain may have seemed strange indeed. Again, the Celts usually went by the name of *Cymri*, not very unlike Homer's *Cimmerii*. Lastly, in the fifth century A.D. Procopius relates explicitly a legend according to which the souls of the dead sail in ships from the Gallic coast to an island called *Britia*, which he assumed lay somewhere north of Scotland, and was inhabited by Britons, Saxons and Frisians.

Such are the data, or "circumstances." The hypothesis suggested is that originally Britain was really regarded as the island of the dead. By the fifth century A.D., however, Britain was sufficiently well known to render this identification impossible, and so Procopius located the "island of the dead" farther north. Whether the hypothesis is true or not need not be discussed here. The main point is that the hypothesis gives meaning to a number of things which would otherwise appear to have no significance.

§ 2. *Circumstantial Evidence and Generalization.*

It was pointed out above that in numerous cases, perhaps in the vast majority of cases, in which people reason from circumstantial evidence no attempt is made to generalize the result. That is so because the cases mostly dealt with are such as are not likely to recur in precisely the same form, and if cases should arise which are partly, but only partly, similar, then recourse is had to analogy, at

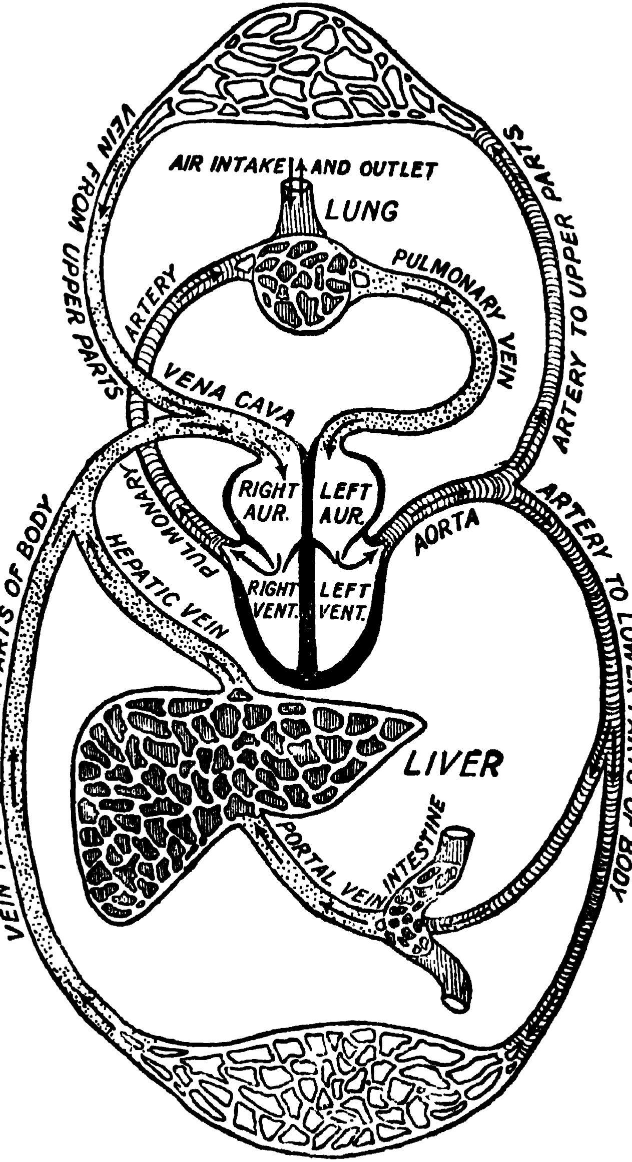
least as a jumping-off board. There are, however, cases in which reasoning from circumstantial evidence is applied to facts which are not unique but actually typical of innumerable similar facts. In such cases the reasoning from circumstantial evidence is at once generalized, the conclusion arrived at being assumed to be true of the whole class of phenomena in question. This is fairly common in science. For example, Harvey's discovery of the circulation of the blood was really arrived at by reasoning from circumstantial evidence. The main data were these. The heart by its alternate contraction and expansion acts like a pump. As the heart contracts so the arteries expand, and within the heart itself the ventricles contract when the auricles do so. It looks therefore as if the blood is pumped from the auricles to the ventricles, and from the ventricles into the arteries. Again, the blood in the arteries can only flow in one direction, namely, away from the ventricles, because valves prevent it from flowing in the reverse direction. On the other hand, the blood in the veins can flow only towards the auricles of the heart, because valves prevent it from flowing in the reverse direction. Moreover, the amount of blood which the ventricles send into the arteries in the course of a single hour is more than the whole body can hold. Where, then, does the incessant flow of blood go to, and where does it come from? Considering the whole structure of the blood-vessels, the only plausible answer is that the blood flows from the arteries into the veins, from the veins into the auricles of the

heart, and from there into the arteries again. And this, stated very sketchily, is what is meant by the circulation of the blood. (See the Diagram and description at the end of this section.) Obviously, it was reasoning from circumstantial evidence that led to the mental construction of the interconnection between the various parts concerned so as to conceive them as parts of one vascular system. But this interpretation can, of course, be applied wherever a similar anatomical structure is to be found. The conclusion, though reached by reasoning from circumstantial evidence, is therefore general in character, or a generalization. And this is true of a good many scientific inferences. It would be no great exaggeration to say that the application of most inductive methods consists, in the first instance, in reasoning from circumstantial evidence, and then generalizing. But of this something more will be said in due course. Here we need only add the remark that the ultimate goal of all human reasoning is to obtain a synoptic view of the drama of cosmic events, and this ideal aim, in so far as it can be achieved at all, can only be attained, even remotely, by reasoning from circumstantial evidence.

The term "reasoning from circumstantial evidence," which has been used throughout this chapter, obviously draws attention to the kind of evidence from which this sort of reasoning sets out. It does not characterize in any way the type of conclusion at which it arrives. Yet in some ways it is at least as interesting to note the kind of conclusion in which the process ends as the kind of evidence with which

it begins. Now, as already explained, the characteristic feature of this kind of reasoning consists in its construction of a *system*, of which the various items of evidence form coherent parts. For this reason it is both natural and usual to call reasoning from circumstantial evidence "systematic" reasoning, and this name was actually used in Chapter XI, § 4, where systematic reasoning was contrasted with linear reasoning. The one drawback to the use of the expression "systematic" for this purpose is its familiar use in a rather different, though not entirely different, sense.

Diagram to Illustrate the Circulation of the Blood (from *The Discovery of the Circulation of the Blood*, by Charles Singer, published by G. Bell & Sons). As the left ventricle contracts the blood is forced through the valves into the great artery called the *aorta*. From there it passes into smaller arteries and the capillaries until it enters the veins, and then passes through the great vein called the *vena cava* into the right auricle. When this auricle contracts the blood is forced through the valves into the right ventricle, and passes through the pulmonary artery into the lung. From the lung the blood passes gradually into the pulmonary vein, from there into the left auricle, and then into the left ventricle again, and the whole circulatory process is repeated.



AIR INTAKE AND OUTLET
LUNG
VEIN FROM UPPER PARTS
ARTERY TO UPPER PARTS
ARTERY
PULMONARY VEIN
VENA CAVA
RIGHT AUR.
LEFT AUR.
RIGHT VENT.
LEFT VENT.
PULMONARY
AORTA
HEPATIC VEIN
ARTERY TO LOWER PARTS OF BODY
VEIN FROM LOWER PARTS OF BODY
LIVER
INTESTINE
PORTAL VEIN

CHAPTER XVII

CLASSIFICATION AND DESCRIPTION

§ 1. *Classification.*

Science, like all human knowledge, begins with sense-experience. But sense-experience is so diverse and so complicated as to appear almost chaotic. In a real chaos life, or at least a rational life, would be impossible. So from earliest times the human mind sought out the elements of order in the world, and the first step in this direction consisted in the noting of similarities between things. Such noting of similarities between things constitutes an implicit, if not an explicit, classification of them. At first, no doubt, classification subserved strictly practical purposes. Similar objects, or events, were simply such as could be treated as equivalents one for another for certain practical purposes. In course of time, however, as human knowledge gained some freedom from the bonds of immediate needs, increasing disinterestedness was shown in the similarities observed. Classification then tended to become more and more objective, or more and more natural, attention being paid more to the character of the things themselves, instead of to their human uses.

The vast number of classifications spontaneously made by early man is obvious from the evidence of language. Every name expresses the recognition of a class of objects; and language is much older than science. Many of these early classifications were based on inadequate observation. Superficial

resemblances often succeeded in concealing deeper differences, or superficial differences succeeded in disguising more important similarities. Deliberate reflection and scientific study have, therefore, ample opportunity to correct the classifications which are implicit in language. For example, in popular language and thought a whale is just a fish because it lives in water, coal is just a mineral because it is found in a mine, and a sea-anemone is a vegetable because it looks rather like it. For science, on the other hand, a whale is a mammal, coal consists of fossilized plants, and a sea-anemone is an animal. The idea that a gas might be a metal, or that the processes of breathing, burning, and rusting belong to the same class of events, would be altogether beyond popular conceivability, but it is the idea put forward by science. Notwithstanding such differences, however, the popular classifications implicit in language usually form the starting-point of scientific classification. Scientific classification hardly ever begins from the beginning, but rather sets out from current classifications. Sometimes even men of science may feel uncertain about the proper place of certain things in the recognized schemes of classification, and, in that case, it may be best to admit a new class. That is what happened, for instance, in the case of the single-celled creatures now known as *Protista*. Some biologists classed them with vegetables, others with animals, but eventually they were recognized as a distinct intermediate class, and called *Protista*.

Classification, then, is in some ways the earliest

and simplest method of discovering order in nature. To recognize a class is to recognize the unity of essential attributes in a multiplicity of individual instances. Classification is thus a recognition of the one in the many. The method of classification is the first method employed in every science. Long before there is that deeper insight into facts, which is required for the more advanced methods of science, the method of classification can be, and has to be, employed. Many sciences, indeed, remain for a long time in a merely classificatory stage, and have consequently come to be known as classificatory sciences. This is especially true of botany, zoology, and ethnology, and was at one time almost equally true of chemistry, mineralogy, and some other sciences.

Classification is a method of science, it is a way of knowing or regarding things. It is primarily an intellectual activity, not a physical activity. The classification may be exemplified or illustrated by the grouping of objects in a museum, for instance. But the physical grouping is not the real, essential classification: it is only an illustration of it. The essence of a classification consists in the fact that certain things are thought of as related in certain ways to one another. The things may be, and usually are, too numerous to be physically grouped together, and, even if an actual physical arrangement were possible, such an arrangement would be based on a prior intellectual classification, and would not itself constitute a real classification. Classification is a mode of knowledge, a way of

grasping the unity of certain things, and the relation between various kinds of things. Classification has, of course, an objective basis in the actual kinship, or similarity, of the objects classified. The man of science is not supposed to invent or to create, but only to discover the sameness or similarity of character in the things, processes, etc., which he classes together. But sameness of character is something very different from a physical grouping together of objects. The arrangements seen in a botanical garden, or in a zoological garden, or in a natural history museum, or in a museum of ethnology or of mineralogy, are not classifications in the strict sense of the term, but arrangements *illustrating* classifications.

The aim of scientific classification is to see things according to their actual objective relationship. Such a classification is what is meant by a *natural* classification. There are other classifications. It has already been pointed out that the earliest classifications tended to be based on man's practical needs in relation to the objects concerned. Even long after that stage was passed, classifications were, owing to insufficient knowledge, based on merely superficial, or less important, attributes. The history of botany, for example, is, to a large extent, the history of various attempts to classify plants on the basis of all kinds of attributes, such as the character of the leaves, of the fruit, or of the corolla, of the calyx, or of the stamens. What is sought after in natural classifications, especially in biology, is that the things as a whole should be taken into

account, with all their important attributes. To achieve this satisfactorily a great deal of knowledge is usually required. Often, indeed, such knowledge can only be acquired with the aid of the higher scientific methods. The method of classification, therefore, although it is the first and the earliest method of science, may also, in a sense, be the last method of science, for the final outcome of the application of other methods of science to certain classes of facts may be a new classification of those facts. This may be seen to some extent in the recent history of biology and of chemistry.

Classification is not only of individuals into classes, but also of classes into wider or higher classes, and of those into still higher classes. In so far as the ideal is attainable, the facts investigated in a science can be conceived as members of a perfectly orderly scheme of things. All classifications are based on the presence or absence, or the presence in varying degrees, of certain attributes; and those classifications are the most natural in which the attributes selected as the bases of the classifications are such as carry with them the presence or absence, or the presence in varying degrees, of other attributes. Mammals, for example, are usually classified according to the character and arrangement of their teeth, because agreement and difference in these respects are found to be correlated with agreement or difference also in other respects. For similar reasons the classification of the chemical elements is based on their atomic weight, with which their specific heat, also their

boiling point and melting point, are usually correlated, at least within the same periodic group of elements.

Classifications made for special, practical purposes are usually called *artificial* classifications. They are not made from the standpoint of the objects themselves, so to say, but from the standpoint of the practical needs of man. For example, the usual classification of plants, found in standard treatises of scientific botany, is a natural classification. It is based on what is believed to be the objective or natural kinship of the plants themselves and is not intended merely to serve some practical purpose of man, except to satisfy his desire for knowledge, for pure science. But the druggists' and herbalists' classification of plants is different. These classifications have reference primarily to the needs of man for medicinal remedies. Such an artificial classification is perfectly legitimate, and in a sense even natural, namely, in the sense that it is based on *some* objective character of the plants, and that it is suitable for the purpose in view. But it is artificial, and not natural, in so far as the basis on which it rests is in the main something extraneous to the plants themselves.

The more restricted the purpose for which the classification is made, the less informing is it likely to be about the essential objective character of the objects so classified. An extreme case of this may be cited from *Punch*. An old lady went on a railway journey with a menagerie of pets. The railway porter told her what the fare would be for

her dogs, but did not know the tariff for her other pets, so she sent him to the station-master to inquire. When he returned he said: "Station-master says, mum, as cats is dogs, and rabbits is dogs, and so is parrots, but this 'ere tortoise is a hinsect, so there ain't no charge for it." From the limited view point of the railway company's schedule of fares, dogs and cats and rabbits and parrots all belong to the same class. This classification was not made in the interests of science, and what is done for a special purpose can only be judged in the light of its suitability for that purpose, and not by the impersonal standard of pure science.

§ 2. *Description, General and Statistical.*

Classification is intimately connected with description. When objects are recognized as forming a class, the class has to be named and described. The name and the description help to make permanent the result of the process of classifying. They facilitate future reference to the subject on the part of the discoverer, and they make it possible to communicate the discovery to others. Science is essentially the result of co-operation. Scientific workers in any field of inquiry keep in touch with one another through the medium of scientific societies, scientific periodicals and other publications. They exchange ideas and check one another's results. In this way what is likely to be true for all is sifted from the mistakes of the individual.

Now, description may be comparatively easy and simple, or it may be difficult and complicated. In

a simple case a statement of easily recognizable parts, qualities and processes might suffice; in a more difficult case the parts, qualities and processes may be hard to describe, and precise quantitative considerations may be involved besides. The sciences have, accordingly, developed nomenclatures (or systems of names for all the classes of objects with which they are concerned), and terminologies (or systems of expressions, including names, verbs and adjectives, for the parts, qualities and processes of the individual objects included in the various classes), and certain statistical devices for the most convenient and most informing expression of the quantitative aspects of the things within their domain. The terminological schemes and the statistical methods are important aids to description. The description is, of course, a description of things; but the concise, economic description of the things in a class constitutes the definition of the name of the class.

The description of the objects included in the same class will naturally confine itself to attributes which they have in common, and only to some of these. The attributes selected to be included in the description will be those that are considered to be the most important, in the sense that they actually are, or are likely to be, correlated with more of the remaining attributes than are the others. With the progress of knowledge our estimate of the relative importance of the different attributes may change, and the descriptions or definitions are, therefore, liable to revision. When, as frequently

happens in geometry, there are several equally correct ways of describing concisely the same class of objects, then the selection of one of them in preference to the others may be guided by considerations of convenience. That description or definition will be preferred which enables the reader, or listener, to realize most easily the nature of the objects described. That is the reason why in geometry the different classes of rectilinear figures are described by reference to the sides, and not by reference to the angles, although the names of some of them actually have an obvious etymological reference to the angles rather than to the sides, for example, triangle, rectangle, pentagon, etc. In addition to such concise descriptions, which are used in the scientific definitions of the names of the classes, there are other descriptions in use as well; also typical pictures and diagrams, when possible, in order to convey a vivid idea of the kind of thing described, even to those who have never actually perceived an instance of it.

Science, however, aims at exactness, and is not satisfied with anything that is more vague or indeterminate than is necessary. Now no two things are exactly similar, and, in order to place objects at all in the same class, many individual differences have to be ignored, emphasis being laid on the common attributes. But that does not yet get over all the difficulties. Things belonging to the same class may be of the same kind, in the sense that they bear a general resemblance to one another in important ways, yet they may vary, nevertheless,

from one another even in respect of some of these similar features. This is especially true of living objects. Plants and animals of the same species vary from one another in sundry ways. Such variations are not something abnormal, but something quite common, in biology. The exact description of the class must consequently take cognisance of these variations, as well as of the resemblances. An example or two may help to elucidate this.

Prawns have dorsal teeth, but the number of dorsal teeth varies from individual to individual. Some have only one dorsal tooth, while others have as many as seven dorsal teeth. How shall the type be described with reference to the number of these dorsal teeth? To answer this question a biologist examined 1,434 specimens, which he collected from an estuary near Plymouth. The examination of their dorsal teeth gave the following results:—

2	had	1	dorsal	tooth
23	,,	2	,,	teeth
103	,,	3	,,	,,
533	,,	4	,,	,,
681	,,	5	,,	,,
89	,,	6	,,	,,
3	,,	7	,,	,,

In a case like this the type might be described by means of some kind of *average*. One might take an arithmetical average or *mean*, adding up the number of teeth possessed by all the above prawns together, and dividing the total by the number of prawns. The typical number of dorsal teeth would then be about 4·5. Or one might take the *mode* as typical,

that is the number which is found most commonly, in this case it would be five, which is the number of dorsal teeth possessed by the members of the largest group. Or one might regard the *median* as typical, that is the value of the individual in the middle of the whole collection, when all the individuals in the collection are arrayed, or conceived to be arrayed, in an ascending or descending order of magnitude. In this case the middle prawn would be the 717th or 718th, and both fall within the group of the 681 with 5 dorsal teeth.

Take another example, clover sometimes has flowers in which one or more florets are higher than the rest. De Vries examined 630 specimens with the following results:—

325	clover flowers had	0	raised	floret
63	,, ,,	1	,,	,,
66	,, ,,	2	,,	florets
51	,, ,,	3	,,	,,
36	,, ,,	4	,,	,,
26	,, ,,	5	,,	,,
18	,, ,,	6	,,	,,
7	,, ,,	7	,,	,,
6	,, ,,	8	,,	,,
1	,, ,,	9	,,	,,
1	,, ,,	10	,,	,,

In this case the arithmetic average or mean would be about 1·5, while the mode and the median would be 0.

When the type has been determined in one of the above ways it still remains to indicate the extent to which the individuals deviate from the type. The individuals in a class of a given average, or type,

may be more or less homogeneous, and the extent of their homogeneity or heterogeneity must be indicated in some way. This is done by measuring the deviation from the average, in simple collections like the above, and ascertaining the typical or average deviation from the average or type. In the case of the prawns, for instance, one would tabulate the differences between five and the actual number of dorsal teeth possessed by all the individual prawns examined, and take either the arithmetical average of these differences, or deviations (called the *average deviation*), or their median (called the *median deviation* or *probable error*). The prawns show a median deviation of one; the typical number of dorsal teeth would, therefore, be represented as 5 ± 1.

The raised florets of the clover-flowers have a median error (also an average deviation) of about $1 \cdot 5$, the same as their arithmetic mean, so that the typical number of raised florets would be expressed by $1 \cdot 5 \pm 1 \cdot 5$.

Another kind of average deviation frequently employed is that known as the *Standard Deviation* (or σ). It is the square root of the average of the squares of the deviations from the arithmetical average of the group. And the expression "*probable error*" (p.e.) is sometimes used conventionally for the standard deviation multiplied by a constant.

§ 3. *Classification and Other Methods.*

Classification is not only the earliest and simplest method of science in the sense already explained,

namely, that it is the method applied even when no other can be applied as yet, it is also the basic method of science, in the sense that its results are usually assumed when the other methods are applied. For example, whether the biologist is endeavouring to establish an evolutionary development between certain remains of animals, etc., or whether the physicist tries to establish a causal connection between certain physical phenomena, what he is always thinking of is not the individual objects before him as such, but the whole classes or types of objects of which they are specimens. As already explained, science is not as a rule concerned with individual objects or instances as such, but with types or classes. This means usually that the investigator assumes that the method of classification has been accurately applied in arriving at the classification of the phenomena with which he is at the moment concerned. Of course, the same instance, or individual object, or event, can be variously classified according to its various characteristics. And the validity of the generalization based on one or more instances will, apart from other conditions, depend on the accuracy of the class-concept under which the instance or instances have been subsumed. Many popular prejudices are rash generalizations on the lines of irrelevant or illegitimate class-concepts. Care is obviously necessary.

§ 4. *Description and Definition.*

There is one kind of description which deserves special consideration. It is familiar under the name

of Definition. Definition may be described generally as a concise description of a class of objects, real or conceivable. The results of a process of classification are secured and made permanent by giving a name to every class, and defining the names. The main function of dictionaries is to keep a record of names and definitions so obtained, and to make them readily accessible. There are two kinds of definition, *genetic* and *substantial*. (*a*) A *genetic definition* is one which describes the mode of generation or production of the kind of thing whose name it defines. Thus, for example, a circle may be defined as the figure traced out when one end of a straight line is fixed while the other is revolving. Similarly a sphere may be defined genetically as the figure described when a semicircle is revolved round its fixed base (the diameter of the circle); and so on. Most chemical formulæ may be regarded as such genetic definitions. Genetic definitions, however, are not so common as substantial definitions. For the most part people want to know what kind of a thing it *is* that the name stands for, rather than how it is *produced*. (*b*) A *substantial definition*, then, is a concise description of what sort of a thing it *is* that the name defined stands for, or means. The simplest way of stating this is to give the name of the next wider or more comprehensive class of which the things in question constitute a sub-class, and to state the characteristic or characteristics which distinguish the sub-class in question from cognate sub-classes. Now, in the wider or popular sense of the terms (as distinguished from their

strict biological use) any wider class consisting of sub-classes is said to be the *genus* of those sub-classes, while the sub-classes are called the *species* of that genus; and the characteristic or the characteristics which distinguish one species from other species of the same genus (or cognate species) is called the *difference* or *differentia*. Again, the Latin for "nearest" is *proximum*. Hence this kind of substantial definition is commonly known as *definition per genus proximum et differentiam*. Thus, for instance, suppose the term to be defined is the term "rectangle," then the next wider class of the objects referred to is "parallelogram," and the "difference" which distinguishes "rectangles" from other parallelograms is that the former are right-angled. The definition of "rectangle" will therefore be "a right angled parallelogram." Similarly a "square" may be defined as "an equilateral rectangle," and so on.

If a substantial definition does not strictly follow the rule of "*the nearest genus and the difference*," it will either be wrong or misleading. Thus, for instance, to define "square" as "an equilateral parallelogram" is wrong; the definition includes other figures than squares, because instead of giving the *nearest* genus (rectangle) it gives a remote genus (parallelogram). Again, to define "equilateral triangle" as "a triangle having all its sides and angles equal" is also objectionable because it is misleading, for anybody not acquainted with the subject might be led to suppose that triangles may have equal sides without also having equal angles, or *vice versa*. When one characteristic of a class of

things involves another (as is the case with the *equilateralness* and the *equiangularity* of triangles) it is obviously unnecessary to state *both* in a succinct description, and, as just explained, it may even be misleading to do so, instead of being more helpful. Such a characteristic implied by another characteristic already included in the definition is called a *proprium* (or *property*, in this technical sense) of the term defined. Any quality not regarded as essential to the class of objects whose name is defined is called an *accidens* (or *accident* in this technical sense), and should never be included in the definition of the name.

§ 5. *Classification and Division.*

Classification is usually regarded as the processes of "ordering" things, that is, of mentally arranging them, according to their similarities and differences, into classes of individuals, and classes of classes. Individual objects or events, etc., are the starting-point for the method of classification, and the result of its application, or continued application, is a class, or a series of increasingly wider classes—or classes and sub-classes, or genera and species, as they are commonly called. The widest class of objects studied in any science is usually called the *highest genus* (*summum genus*) for that science, while the most restricted sub-class which cannot be usefully divided into other narrower sub-classes, but only into individuals, is known as the *lowest species* (*infima species*). Classification may therefore be said to proceed from individual objects to lowest species, and

thence to higher and higher genera until it reaches the highest genus for the science in question.

Now *Division* is the name of the reverse process of beginning with some generic class, and distinguishing within it various sub-classes, each of which may in turn be again sub-divided into other sub-classes, until the lowest species are reached which cannot, to any purpose, be further sub-divided except into individuals.

The *results* of classification and division are essentially the same, namely, a scheme of inter-related classes. From the tabulated scheme it would often be difficult, if not impossible, to tell whether the scheme was obtained by continued classification, or by continued division. It is only the *direction of the process* that is different in the two cases. In classification the direction is *from* individuals towards the *summum genus*; in division it is reversed. The ultimate result is essentially the same. Only in some cases, especially in cases of ideal or mental constructions, such as geometrical figures, types of inference, etc., it is simpler to proceed by division, in others, such as most empirical phenomena, it is impossible to proceed otherwise than by classification mainly. At the same time it would be a mistake to draw a hard and fast line between the two processes even as processes, for, in the case of many so-called classifications, classification and division are more or less combined, some distinctions having been learned from experience pure and simple, while others are suggested by logical or rational considerations

CHAPTER XVIII

THE EVOLUTIONARY AND COMPARATIVE METHODS

§ 1. *The Evolutionary or Genetic Method.*[1]

There are some facts which are not sufficiently similar to be regarded as belonging to the same immediate class, but are similar enough to make us regard them as belonging to neighbouring classes, or as sub-classes of some higher class. In such cases one is sometimes led to suppose that the similar classes are kindred in the more or less usual sense of being descended from a common ancestor or antecedent. A new problem thus arises, namely, that of tracing the stages in the descent or the development of the kindred classes. This is especially true of living objects, their organs, and their functions, etc. In the case of human beings, there are similar problems relating to the origin and the development of their customs, their institutions, and their inventions, etc. The scientific method by which such developments are traced is known as the Genetic Method, also as the Evolutionary Method; and in so far as the method succeeds in establishing the stages in the evolution of certain classes of facts, a better insight is obtained into their unity and

[1] This method is frequently called the Comparative Method (see the next section). Sometimes it is also called the Historical Method. But this name is so ambiguous that it is best to avoid it. Mill applied it to a form of the Deductive-Inductive Method (see Chapter XXI). And not infrequently a problem is said to be treated by the Historical Method when all that is meant is that an account is given of the different *views on the problem* put forward by various investigators at different times.

continuity than is afforded by ordinary classification. Sometimes, in fact, the real significance of the resemblances on which the usual classifications are based, or to which they draw attention, is first brought out by the Evolutionary Method, which elaborates a mere table of similar classes into a genealogical tree, develops a merely static classification into a kinetic or phylogenetic scheme. The various classes are then unified or connected very intimately by being shown to be phases or stages of one continuous process, or a system of intimately connected processes.

The science which appears to have been the first to employ the Evolutionary Method is Comparative Philology, which used it already in the eighteenth century. The ground was prepared for its use in Comparative Philology, inasmuch as people were long familiar with the idea of the unity of mankind, and the existence of a common human language until the time of the Tower of Babel, since when that ancestral tongue, it was believed, had developed into a variety of languages. It was, therefore, felt to be reasonable to compare the different languages in existence, and to attempt to trace the history of their evolution in the light of such similarities and differences as the comparisons disclosed. This very assumption of an historic development in the case of languages constituted the first requisite for the application of the Evolutionary Method in Comparative Philology.

It was different with Comparative Anatomy. The same Biblical narrative which had facilitated the

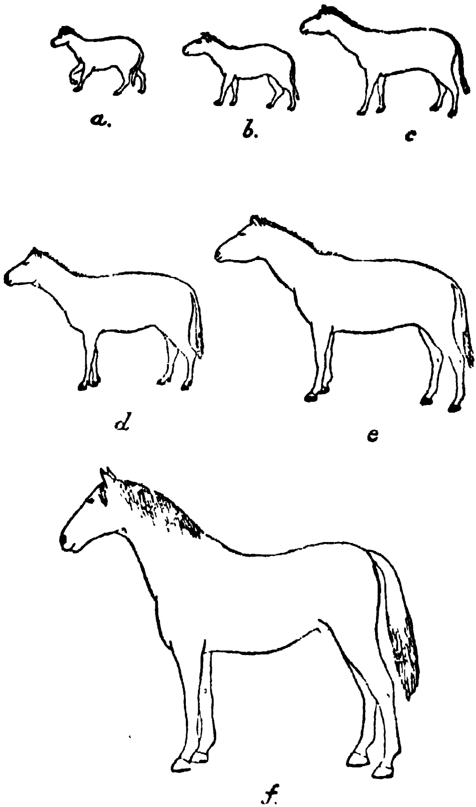
application of the Evolutionary Method in Comparative Philology hindered its application in Comparative Anatomy. For it taught, or was supposed to teach, that the different kinds of animals had each been created separately, and were thus distinct from one another in their origin. The similarities observed by the early anatomists were, consequently, regarded merely as interesting curiosities, and led no further until the time of Darwin, whose *Origin of Species* is the classic application of the Evolutionary Method in Biology. Only since the publication of this book has Comparative Anatomy really become comparative in the sense of the Evolutionary Method.

The Evolutionary Method is, then, applicable only to those classes of facts which can, tentatively at least, be regarded as the products of a process of development. It is the function of the method to indicate (*a*) the main steps or stages through which the development has probably taken place, and (*b*) the reason for the various changes constituting the several stages in the suggested line of development. When all the stages of the evolution of anything are known from direct observation and record (as in the case of many varieties of fruits, of pigeons, or the stages in the development of the bicycle), there is no occasion to apply the Evolutionary Method. It is only in cases where few of the earlier forms are known that the Evolutionary Method proceeds hypothetically to suggest a probable line of development. Thus, for example, in the case of the horse we have no such detailed

knowledge of its descent from a five-toed ancestor as we have of the varieties of pigeons descended from the wild wood-pigeon. Only a few of the alleged intermediate forms of the horse are known, and the Evolutionary Method, basing itself on such evidence, has put forward an hypothesis relating to the probable series of variations through which the horse, as we now know it, has passed.

The whole theory of biological evolution rests on applications of the Evolutionary Method; and all the phenomena to which the conception of evolution is applicable afford opportunities for the application of that method. The method can be applied, and is, indeed, being applied, not only to plants and to animals, to social customs and social institutions, to the human mind, to human ideas and ideals, but also to the evolution of geological strata, to the differentiation of the chemical elements, and to the history of the solar system.

When searching for the gradations through which some product of evolution has passed, the correct thing, according to Darwin, is to look chiefly among kindred classes of objects (animals, organs, etc.). But it is rarely possible to obtain sufficient evidence from a study of the nearest kindred only, and the investigator is, therefore, frequently compelled to go farther and farther afield, among less and less kindred classes of objects, in search of missing links in the chain of evolution. Thus, for example, Darwin himself, when dealing with the development of the honeycomb of the hive-bee, kept close to kindred species, but, when he traced the evolution

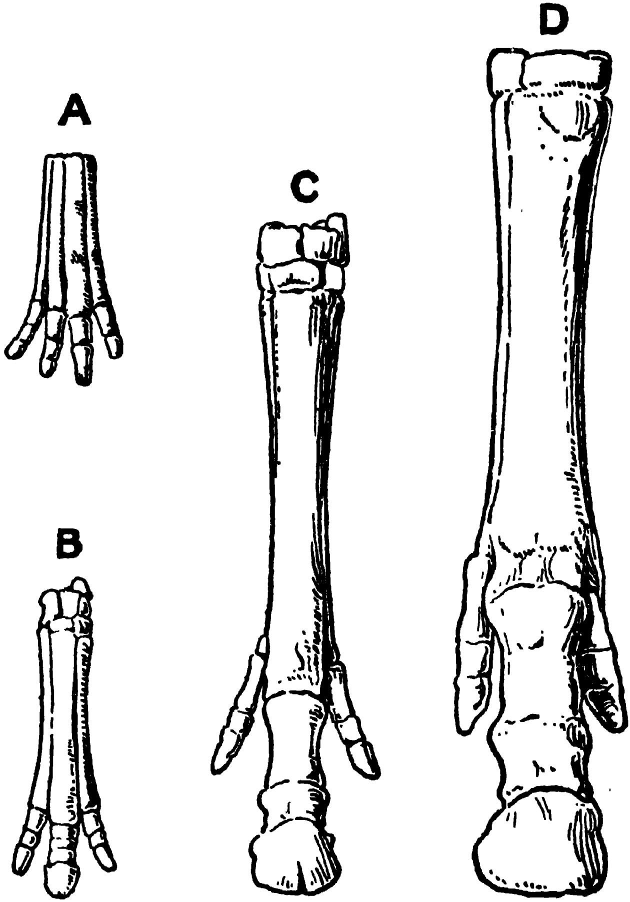


THE ANCESTORS OF THE HORSE AND ITS RELATIVES COMPARED IN SIZE AND FORM WITH THEIR TYPICAL MODERN REPRESENTATIVE.

a. *Hyracotherium*, or *Protorohippus*, of the Lower Eocene; *b*. *Plagiolophus* or *Orohippus*, of the Middle Eocene; *c*. *Mesohippus*, of the Oligocene; *d*. *Merychippus*, of the Miocene; *e*. *Pliohippus*, of the Pliocene; *f*. The Modern Horse, *Equus caballus*, domesticated breed (Lull, *Amer. J. Sci.*, vol. xxiii, p. 167).

of the eye of vertebrate animals, he went far afield, referring to facetted eyes, eyes without a lens, and eyes which are mere collections of pigment cells. Similarly, the comparative psychologist, when tracing the evolution of the human mind, does not confine himself to the primates, or higher apes, but seeks light also among the lower animals.

As an example of the Evolutionary Method we may refer again to the development of the horse. The present-day horse is a large quadruped, has only one toe on each foot, a splint-bone on each side of the upper end of the cannon-bones, and so on. The fossil remains of quadrupeds now extinct have enabled zoologists to reconstruct the history or evolution of the horse, which the accompanying sketches may illustrate. In the Lower Eocene Age there existed a small kind of quadruped not larger than a fox, in some ways much simpler in structure than the modern horse, but having four toes on each foot. In Oligocene times the descendants of these quadrupeds had only three toes on each foot, but were larger in body. In the course of time the middle one of the three remaining toes gradually increased in size, and was alone used in walking and running, while the other two toes became smaller and smaller, eventually remaining merely as splint-bones. Concurrently with these and other changes, the body of the horse grew in size. More speculatively, the history of the horse is carried a stage farther back to a still smaller ancestor that had five toes on each foot, and walked on the whole sole of its foot instead of on one toe only.



SKELETON OF FORE-FEET OF EXTINCT FORE-RUNNERS OF THE HORSE:

A. *Hyracotherium* (No. N. H. 65); B. *Mesohippus* (No. N. H. 63); C. *Merychippus*, or *Protohippus* (No. N. H. 57); D. *Hipparion* (No. N. H. 44).

[*Guide to the Horse Family*, British Museum (Natural History).]

§ 2. *The Comparative Method.*

The term "Comparative Method" is frequently or even usually employed as synonymous with the term Evolutionary Method, explained in the foregoing section. This use of the term is, on the whole, warranted. It has come about in this way. Some sciences have long been known as "Comparative Sciences"— Comparative Philology, Comparative Anatomy, Comparative Physiology, Comparative Psychology, Comparative Religion, etc. Now the method of these sciences came naturally to be described as the "Comparative Method," an abridged expression for "the Method of the Comparative Sciences." And when the method of most comparative sciences came to be directed more and more to the determination of evolutionary sequences, that is to say, became evolutionary in that sense, the term "Comparative Method" came to mean what is now frequently described as the "Evolutionary Method."

The method of the comparative sciences, however, was not always the Evolutionary Method, and is not always so even now. And, in consequence of certain differences among sociologists and ethnologists, the tendency is to distinguish between the Comparative Method and the Evolutionary Method, the latter term being employed in the sense explained in the previous section, while the former is given another meaning. The precise meaning of the term Comparative Method, when it is distinguished from the Evolutionary Method, is not easily determined, as it appears to be used somewhat

loosely and nebulously. To say that the Comparative Method is a method of comparison is not illuminating, for comparison is not a specific method, but something which enters as a factor into every scientific method. Classification obviously requires careful comparison; and every other method of science depends upon a precise comparison of phenomena and the circumstances of their occurrence. All methods are, therefore, "comparative" in a wide sense. How, then, does the term Comparative Method come to be used at all in a wide sense, as distinguished from its restricted meaning when it is regarded as synonymous with Evolutionary Method?

The answer is to be found partly in the somewhat special or peculiar circumstances of Sociology, the science of social groups. The most familiar way of studying a social group is that of the historian. Now, the historian gives a chronological account of individual social groups as such; *qua* historian it is not his business to compare a number of social groups with a view to generalizing about them. Sociology, however, being a science, and not a history, is concerned with the discovery of general truths relating to social groups. The sociologist, accordingly, does compare many social groups, to note their similarities and their differences; he even studies what is known of extinct social groups, or of extinct customs and institutions. In order to stress the *general* character of his study (in contrast with the *particular* character of the strictly historical study of a social group), he describes his science as

a *comparative* study of social groups or institutions. Now, the comparison may lead to classification, the classification, for instance, of the main types of social structure, or of the principal forms of human marriage. It may lead to the application of the Method of Agreement (see Chapter XIX, § 5) in order to establish some causal connection, as, for example, when a comparison of the various circumstances under which the practice of human sacrifice is met with leads Professor Westermarck to the conclusion that the motive prompting it is that of life-insurance, based upon the idea of substitution. Or, again, the comparison may lead to the application of the Joint Method of Agreement and Difference (see Chapter XIX, § 7), as, for instance, when Dr. Lowie tries to establish a connection between the social system of clan exogamy and the classificatory system of relationship. The other inductive methods may be similarly employed in consequence of such comparisons. And sometimes the comparison of social institutions at different stages of culture may lead to the discovery of an evolutionary sequence or series, and the application of the Evolutionary or Genetic Method. But, it is argued, the comparative study of any such phenomena is not necessarily bound up with the tracing of evolutionary development. A comparative study of phemomena may be pursued by investigators who do not believe that the phenomena in question are the products of evolution, or who possibly do not believe in evolution of any kind. Hence the need of differentiating between the Comparative Method and the Evolutionary Method.

The foregoing considerations may render intelligible the term Comparative Method as it is sometimes used. This usage, however, is not to be commended. For, according to it, the term Comparative Method is little more than a vague name for *any* scientific method. The main purpose of the sociologists concerned would be attained more accurately, and at least as effectively, if they simply distinguished between *Special* Sociology and *General* Sociology. This would correspond to the common distinction between, say, special anatomy (e.g. human anatomy), or special philology (e.g. English philology), on the one hand, and comparative anatomy, or comparative philology, on the other. In the so-called "comparative" sciences the word "comparative" is probably too well established to be abandoned; not so in Sociology. But even if the term "Comparative" Sociology should be used instead of General Sociology, there is no adequate reason for continuing to use the rather nebulous expression "Comparative Method," instead of specifying the precise methods intended, such as that of Classification, Agreement, etc.

Generally speaking, in so far as the comparative study of biological phenomena does not end in classification, the observation of similarities does not really explain them, but rather calls for an explanation. For example, the resemblances observed by early biologists (for instance, Belon, in his *Book of Birds*, 1555) between the bones of man and the bones of birds, constituted a problem, rather than an

explanation. And one possible way of solving such a problem is by establishing a genetic relationship, or kinship, between the similar phenomena. In that way the Evolutionary Method finds a natural place among the methods of the comparative sciences.

§ 3. *Method, Hypothesis, Working Idea, Theory.*

The formulation of an evolutionary *method* may cause perplexity to some people. They are familiar, more or less, with an evolutionary *hypothesis*, or with a *theory* of evolution, they may say, but as to an evolutionary *method*, why the very suggestion seems to be the result of a confusion between hypothesis or theory, on the one hand, and of method on the other. It may, therefore, be worth while indicating briefly the main distinctions between these terms, in order to prevent, or to remove, unnecessary perplexities.

A scientific *hypothesis*, as already explained, is a very definite suggestion or supposition concerning the relation between certain phenomena; and it is so definite that it can be confirmed, or else confuted, by definite observations. But at the back of hypotheses there are always certain vague *working ideas* or assumptions which cannot be so confirmed or confuted, but which prompt all hypotheses of a certain kind, and without which such hypotheses would never come into being. Now a *method* is a kind of procedure with the aid of which the vague working idea or assumption takes definite shape

as an hypothesis, and by which the hypothesis is tested. An example may make this clear. The belief in the causal connection of physical phenomena is a vague working idea which, as such, is incapable of confirmation or of confutation. On the other hand, the suggestion first made in the seventeenth century, that air is a causal condition of combustion, was a definite hypothesis, which was definitely confirmed by certain familiar experiments. The experiments followed the method of difference, showing that under otherwise similar conditions combustion took place when air was present, but did not take place when air was absent. Now it is under the influence of the general assumption that there are such things as causal connections that there originated the definite hypothesis of a causal connection between air and combustion, and the method of difference was the way in which the definite hypothesis may have been suggested, and certainly was tested. In the same kind of way the vague general assumption that there is such a thing as evolution prompts various definite hypotheses relating to evolutionary connections between specific kinds of objects (such as different kinds of eyes, from mere pigment cells upwards, or different kinds of skulls, or of implements, etc.), and the evolutionary method is the way by which the vaguely general working idea of evolution passes into, and tests, definite evolutionary hypotheses concerning certain specific series of facts. The working idea is one thing, the hypothesis is another, and the method is yet a third thing *Mutatis mutandis* these dis-

tinctions may also be illustrated by reference to the method of classification, and, in fact, to every other method of science.

Again, the term *theory* is used in a variety of senses. Sometimes when people speak of the theory of evolution all that they mean is the general working idea or assumption; sometimes they mean some definite evolutionary hypothesis, namely, that of the evolution of man from non-human ancestry; and sometimes they mean theory in the sense used in this book, namely, a comprehensive generalization based on a number of less comprehensive generalizations already confirmed. In the last sense of the term *theory*, the theory of evolution means the comprehensive generalization that there is evolution throughout the animal, or even the organic, kingdom, and it is assumed that this comprehensive generalization is an induction based on more restricted evolutionary inductions such as were illustrated above. Such a theory is, of course, the strongest confirmation possible of a working idea or assumption. But the mere *working idea* and the *theory* are different *toto cælo*. To identify them would be to identify the beginning with the end, or the highly finished and polished product with its crudest beginnings. And the working idea or assumption, just because of its relative vagueness, is always ahead not only of definite hypotheses proceeding from it, but also ahead of comprehensive theories resulting from such hypotheses. One need only think, for example, of the working idea of evolution as extended tentatively beyond the confines of the

animal kingdom, or of the working assumption of causality as applied in all sorts of directions.

Such working ideas when they are of an ultimate character are usually described as *principles* or *postulates*.

CHAPTER XIX

THE SIMPLER INDUCTIVE METHODS

§ 1. *Classification and Law.*

The scientific search for general truths is satisfied in some measure by the discovery of natural classes through the Method of Classification, and by ascertaining evolutionary sequences with the aid of the Evolutionary Method. In the former case, we discover certain uniformities of co-existence among the groups of essential characteristics of the several natural classes. In the latter case, we discover certain uniformities of sequence among various complex phenomena, which follow one another as successive phases or stages of an evolutionary process. On the strength of our knowledge of a uniformity of co-existence among the attributes of a class, it is possible to infer from the presence of some class-characteristics in an object (e.g. the frontal horns and the hoofs of a quadruped) also the presence of certain other characteristics (e.g. the possession of a ruminant stomach, and graminivorous habits [1]); and on the strength of our knowledge of a uniformity of sequence among the stages of the evolution of certain phenomena, one might anticipate the coming of a subsequent stage from the observation of an earlier one, or imaginatively interpolate something between two known stages.

[1] There is a story that the devil once came to Cuvier, the famous zoologist, and threatened to eat him up. The zoologist looked the devil up and down, and replied: "You can't! You have horns and hoofs! Go and eat grass; you can't eat me!"

But such uniformities as the foregoing are for the most part only empirical, and not altogether satisfactory. Science looks as far as possible for what can be more or less adequately proved. Even the relative values of different classifications will be assessed according to their usefulness for real inductions. Now all the proofs of general truths take one of two forms. One of them is the type with which we are familiar from geometry, where it can be shown by sheer intuition, or by deductive reasoning from intuition, that certain attributes must be present where certain other attributes are present; but the fundamental uniformities of natural science cannot be established in that way. The other method of proving uniformities is by ascertaining either the direct or indirect causal connection between the terms of the uniform relationship. In both forms of proof we try to establish relations between conditions and consequents; only in Mathematics (also in Logic) we are concerned with *rational* conditions (or reasons) and consequents, while in natural science we are concerned with *physical* conditions and results (in psychology with psychical conditions and results which are intimately connected with physical or physiological conditions and results).

The world, as we see it, is a vast complex of incessantly changing things, which the human mind endeavours to grasp by mentally, and sometimes also physically, analysing into simpler constituents, and ascertaining the laws or regularities of their connections, or their correlations, if there be such.

The facts themselves do not manifest their intimate relations with one another. We can only solve the riddle, if at all, by surmising what the relations may be. Such surmises are only fruitful when they have been preceded by close observation of the facts and are followed up by a still more searching observation and (where possible) experimentation.

§ 2. *The Five Canons or Methods of Induction.*

The kinds of observations by which the man of science is led to surmise a real connection between certain facts, and the kinds of observation by which he then proceeds to test his surmise, or hypothesis, are often very similar. Their general character has been formulated in the five so-called Canons of Induction. These are not the only methods of ascertaining laws, or uniformities, or regularities among the phenomena of Nature. We have already described two other methods of doing so, and yet other methods will be explained in due course. But the five canons or methods of induction are important all the same.

The principle underlying these canons is this. If, other things remaining essentially the same, a certain factor or circumstance cannot be omitted, or quantitatively changed, without changing a certain phenomenon, then that factor, or circumstance, is a condition of that phenomenon, or. in other words, is intimately connected with it. Assuming, as we generally do, that things and events are not merely a matter of chance, but are the results of operative conditions, we examine

instances of the phenomenon, in which we are interested, under sufficiently varied circumstances to enable us to detect what it is that cannot be removed or altered without removing or altering the phenomenon in question. Not that the presence of an element of chance in the universe is to be ruled out *ab initio*. We shall return to this point in due course. But it is order and regularity that have helped man most in his struggle for existence. It is order also that satisfies most the rational tendency and the æsthetic sense of man. So we naturally look for order first, and only reluctantly relinquish our search for it when we are baffled in our quest.

Now the process of ascertaining what is indispensable to a certain phenomenon may assume one of two forms, a direct form and an indirect one. In the direct form it is shown by observation or experiment that (*a*) the elimination, or (*b*) the quantitative variation of a certain factor or antecedent is followed by (*a*) the elimination, or (*b*) the quantitative variation of the phenomenon under investigation, although all other relevant factors have remained the same. In the indirect form of the inductive process it is shown that, so long as a certain antecedent remains operative, no change in any of the other relevant circumstances makes any material difference to the phenomenon under investigation, which must, therefore, be intimately connected with the constant antecedent. The first type of the direct form is known as the Method of Difference, the second type of the same form is called the Method of Concomitant Variations. The

indirect form is known as the Method of Agreement. Of the two remaining Canons or Methods, one is known as the Method of Residues, and is really a slight modification of the Method of Difference, while the other is known as the Joint Method of Agreement and Difference (also as the Method of Exclusion, or as the Double Method of Agreement), and is a kind of approximation to the Method of Difference, secured by supplementing the Method of Agreement in certain directions.

The several inductive methods have different degrees of cogency. The Methods of Difference and of Concomitant Variations are the most conclusive. But even these methods cannot always be applied rigorously. When, as sometimes happens, the phenomena under investigation are not sufficiently under the control of the investigator, he may not be able to secure the precise kinds of instances required for the strict application of these methods. But, in science, as in life generally, if one cannot command the best means, one tries the next best, and so on. In such cases, one usually endeavours to strengthen the result of the less strict application of one inductive method by the application of some of the other inductive methods as well, sometimes even by resorting to deductive reasoning from the nature of the case. The separate exposition of the several inductive methods must not be taken to imply that each of them is usually, or should be, employed alone. They are frequently employed in conjunction; and in their less cogent forms it is not always easy to distinguish one from another, say, the

Method of Difference from that of Concomitant Variations or from the Joint Method.

§ 3. *The Method of Difference.*

If two sets of circumstances are alike in all relevant respects except that in one of them (called the Positive Instance) a certain antecedent is present and also a certain consequent, while in the other (called the Negative Instance) both are absent, then that antecedent and that consequent are related as condition and consequent, that is to say, that consequent will always follow that condition. Symbolically, if antecedents a,b,c,d are followed by consequents w,x,y,z, while when d is absent the antecedents a,b,c are not followed by z, then d is a condition of z.

$$\underbrace{a\ b\ c\ d\ \ldots}_{w\ x\ y\ z\ \ldots}\text{[1]}\ ; \qquad \underbrace{a\ b\ c\ \ldots}_{w\ x\ y\ \ldots}\ ; \qquad \text{therefore } \begin{matrix} d \\ | \\ z \end{matrix}$$

For example, suppose a piece of litmus paper when dipped into acid turns red at once, while another exactly similar piece of litmus paper not dipped into acid (but dipped, say, into water or some other liquid) does not turn red, then the acid is a condition of its turning red. Again, suppose a surface exposed to the air, and having the same temperature as the air, is dry, while as soon as the temperature of the surface falls below that of the air, then, although

[1] The dots after the symbols (throughout this Chapter) are intended to make clear that there are other antecedents, and other consequents also present, which, however, are not considered relevant to the problem.

the remaining circumstances remain the same, condensation of moisture takes place on that surface, in that case the lower temperature will be regarded as a condition of the condensation. Similarly, if a healthy animal is inoculated with the blood of another animal suffering from anthrax (splenic fever) and contracts the disease, then the inoculation will be regarded as a condition of the infection. Or, again, if a freshwater crayfish, having its antennules (small feelers) intact, retreats from strong odours, while another, bereft of them, does not react to strong odours at all, then it may be inferred that the antennules are the seat of the organ of smell.

To secure the requirement that only one relevant circumstance should distinguish the two cases compared, it is often necessary to use technical aids. In some cases indeed one could not determine at all the actual influence of one of the factors without technical aid. For example, the weight of air, the influence of air on moving bodies, its function in breathing, burning, and rusting, could not be ascertained at all without the aid of an air-pump. What the air-pump does, however, is to procure for us the requisite kind of instances about which we can reason satisfactorily, on the lines of one or other of these Inductive Methods. In simpler cases we can secure the right instances, to reason about on similar lines, without the aid of technical devices.

The Method of Difference assumes various forms according to circumstances. Sometimes the two

instances compared are really two successive states of the same set of circumstances, to which something is added to obtain the Positive Instance, or from which something is withdrawn to obtain the Negative Instance. On the other hand, the two instances are frequently separate and distinct instances, though similar in all essentials but one. And sometimes, again, the instances compared are not single instances, but are groups of instances, each group being treated more or less as a single instance. The following example may illustrate the group-form of the Method of Difference. When Pasteur tested the efficacy of preventive vaccination against anthrax, which he thought he had discovered, he first vaccinated twenty-five sheep with a mild preparation of the serum, and, when they had recovered, he vaccinated them again, and also twenty-five other sheep, which had not been vaccinated before, with a strong preparation of the serum. The twenty-five sheep which had undergone the preparatory vaccination survived, while the others all perished. This showed that the preparatory vaccination had acted as a protection. The two instances, in this case, were not single sheep but two groups of twenty-five sheep in each, and the result was all the more conclusive, for it is easier to secure essential similarity between two groups, as groups, than between two individuals.

In this, as in every other, form of the Method of Difference, it is most important that the two instances should be as like as possible in all essentials except the difference under investigation. The

neglect of this condition (sometimes called the fallacy of *non ceteris paribus*) easily leads to the mistaken attribution of a result to the wrong circumstance or antecedent (the fallacy of *cum* or *post hoc ergo propter hoc*). The following example may serve as a warning. In a certain hospital in Dublin it was observed that there was a higher rate of mortality among the patients lying in the wards on the ground floor than among those lying in the wards on the upper floors. It was, accordingly, concluded that the ground floor was less healthy than the upper floors. Subsequently, however, it was ascertained that the hospital porter had been in the habit of placing in the wards on the ground floor all patients who were too ill to walk, while those who could walk were taken to the wards on the upper floors.

One of the most important precautions which must be taken in connection with the application of the Method of Difference is this. The introduction of the new factor (" d " in the above symbols) must be so managed that it does not alter in any way in the process of being introduced. Delay is sometimes fatal in this respect. For example, in the course of some experiments in connection with anthrax, two French professors obtained some blood from an animal that had suffered from the disease, and they injected some rabbits with it. The rabbits died rapidly, but did not develop the parasites of anthrax. The French professors thereupon thought that they had disproved the view of Pasteur that anthrax could be communicated in

that way. It turned out, however, that an interval of about twenty-four hours had elapsed between the drawing of the blood from the diseased animal and its injection in the healthy rabbits, and in the meantime the blood had putrified, so that when injected it set up a form of blood poisoning, which killed the rabbits so quickly that the anthrax parasites had no time to multiply sufficiently to manifest themselves.

§ 4. *The Method of Concomitant Variations.*

If an antecedent and a consequent vary concomitantly although no other relevant circumstance has changed, then that antecedent is a condition of that consequent. Symbolically :—

$$\underbrace{a\ b\ c\ d_1 \ldots}_{w\ x\ y\ z_1 \ldots}\ ; \quad \underbrace{a\ o\ c\ d_2 \ldots}_{w\ x\ y\ z_2 \ldots}\ ; \quad \underbrace{a\ b\ c\ d_3 \ldots}_{w\ x\ y\ z_3 \ldots}\ ;$$

[or briefly, $z = f(d)$, i.e. z is a function of d]

therefore $\begin{matrix} d \\ \vdots \end{matrix}$.

The Method of Concomitant Variations is closely related to the Method of Difference, and can easily be expressed in a symbolic form very similar to that of the latter. Let the quantitatively variable antecedent in two instances be represented (as above) by d_1 and d_2 respectively, and the quantitatively variable consequent by z_1 and z_2 respectively, then $d_1 = d_2 + (d_1 - d_2)$, and similarly $z_1 = z_2 + (z_1 - z_2)$. The two instances

can accordingly be symbolized, one as a positive instance, the other as a negative instance, thus :—

$$\underbrace{a\ b\ c\ d_2 + (d_1 - d_2) \ldots}_{w\ x\ y\ z_2 + (z_1 - z_2) \ldots}; \quad \underbrace{a\ b\ c\ d_2 \ldots}_{w\ x\ y\ z_2 \ldots};$$

therefore $\underbrace{(d_1 - d_2)}_{(z_1 - z_2)}$, or the quantitative difference in the antecedent d is connected with the quantitative difference in the consequent z. (Compare the symbolic form in § 3 above.)

The two methods are, of course, not quite the same, even so. In the Method of Difference it is shown that the *presence* of a certain antecedent is connected with the *presence* of a certain consequent ; in the Method of Concomitant Variations it is shown that a certain *quantitative difference in a given antecedent* is connected with a certain *quantitative difference in the consequent.* But the difference between the two methods is not always important.

The tendency of modern science to be quantitatively exact has given special importance to the Method of Concomitant Variations. Over and above its special function to show that there is a connection between certain phenomena in the study of which the other inductive methods are not so helpful, the Method of Concomitant Variations is also applied in all possible cases just to ascertain the quantitative correlation between phenomena, even though their connection is not under consideration—either because the connection as such has already been established, or because it is not believed that there is any direct connection between the

phenomena in question. This use of the method is usually described as *quantitative* induction, in contrast with *qualitative* induction, which merely seeks to ascertain (by means of any of the inductive methods) whether there is any connection at all between certain phenomena, without determining their precise quantitative co-variation. As examples of quantitative induction we may refer to the experiments by which the coefficient of expansion of various substances is determined, or the coefficient of absorption and emission of heat, or the relation between the amount of the radiation and the temperature of bodies, or the mechanical equivalent of heat, or the refractive index of a transparent medium, or the relation between the electric current passing through a conductor and the electric pressure between its ends, and so on. Most, or all, statistical correlations also belong here.

The Concomitant Variation may be direct or inverse. It is said to be *direct* when the antecedent and the consequent increase together, and diminish together. It is said to be *inverse* when one of them diminishes as the other increases. Thus the temperature and the volume of a gas (the pressure remaining constant) are an instance of direct concomitant variation, while the pressure and the volume of a gas (the temperature remaining constant) are an example of inverse concomitant variation. Some phenomena exhibit direct and inverse variations at different stages. For example, the temperature and the volume of water show direct concomitant variation between 4° C and

100° C., but inverse concomitant variation between 0° C. and 4° C. Moreover, the same consequent may be connected with two conditions and vary directly with one of them, and inversely with the other. For instance, the gravitation between bodies varies directly with their masses and inversely with their distances from one another.

Again, as the last example suggests, the concomitant variation (whether direct or inverse) may be simply proportionate, or highly complex. For instance, gravitation varies in simple proportion to the multiple of the masses of the bodies, but it varies inversely with the square of their distances. In some cases the concomitant variation requires a very complicated formula for its expression, and in some instances the concomitant variations are so complex that the correct formulæ for them have not yet been discovered.

The Method of Concomitant Variations is applicable in some cases in which the Method of Difference is impracticable, namely, in all cases in which the conditions studied can be varied in quantity, or intensity, but cannot be eliminated altogether. This applies to heat and gravitation, neither of which can be completely eliminated from material bodies. It is also true of the friction of moving bodies, since no case is known of frictionless motion. Since the amount of friction can be varied enormously, and it is found that the period of motion of a body varies inversely with the amount of friction which it encounters, we conclude that, in the limiting case, if friction could be eliminated

altogether, then a body once in motion would continue to move indefinitely. Thus the first law of motion really rests on the Method of Concomitant Variations.

The following is an interesting instance of the application of the Method of Concomitant Variations. During the cholera epidemics in London in 1849 and in 1854 about a fifth of the population of London was supplied with water by the Lambeth Water Company and the Southwark Water Company. In 1849 both companies obtained their water from the same part of the Thames, so that the water supplied by both companies was equally polluted. The number of deaths from cholera per 10,000 of the population in the area served by the Lambeth Company was 125, and in the area served by the Southwark Company it was 118, that is, nearly the same. In 1854 the Lambeth Company drew its water much higher up the river, where the water was far less polluted, while the Southwark Company retained its old intake. The mortality rate from cholera per 10,000 of the population in the area served by the Lambeth Company fell to 37, while in the area served by the Southwark Company it rose to 130. Evidently the contaminated drinking-water was one of the conditions of the cholera mortality. But polluted drinking-water is something complex calling for further analysis, and subsequent investigation revealed what specific form of water-contamination was connected with cholera. It is not at all uncommon in the history of science that the condition of some phenomenon

is first tracked to a complex antecedent from whi the real condition is then fetched out, as it wei from its hiding-place by a closer scrutiny, that is, l more careful analysis.

In the application of this, or any other, inducti method, inadequate analysis of circumstances m easily mislead one to attribute a result to the wroi antecedent, as the following case may illustrat Dr. W. Farr, an early investigator of the epidemi of cholera referred to in the preceding paragrap discovered an inverse concomitant variation b tween the number of deaths from cholera per 10,0 inhabitants and the elevation of the district which they resided. The following table gives h data :—

Elevation of District in feet.	*Cholera Deaths per* 10,000 *inhabitants.*
Under 20	102
20 – 40	65
40 – 60	34
60 – 80	27
80 – 100	22
100 – 120	17
340 – 360	7

He concluded, accordingly, that "the elevation the soil in London has a more constant relation wi the mortality from cholera than any other knov element." Now, Dr. Farr was on the right scent he was, so to say, getting near the hiding-place one of nature's secrets. The really importa factor, as explained in the foregoing account of tl matter, was a specific pollution of the drinkin water. But at that time London was, to a lar

extent, supplied with water from surface-wells. Now, wells at or about the river level had their waters more polluted than those at a higher level. Moreover, the lower-lying districts of London were more densely populated, and consequently more exposed to infection.

§ 5. *The Method of Agreement.*

If several instances of the occurrence of a phenomenon have one relevant antecedent in common, then that common antecedent is a condition of that phenomenon. Symbolically :—

$$\underbrace{a\ b\ c\ d \ldots}_{w\ x\ y\ z \ldots};\quad \underbrace{b\ d\ f\ g \ldots}_{x\ z\ s\ t \ldots};\quad \underbrace{d\ f\ k\ l \ldots}_{z\ s\ p\ r \ldots};\ \text{therefore}\ \begin{array}{c} d \\ | \\ z \end{array}$$

If dew is deposited on a number of surfaces which are different in all relevant respects except that their temperature is below that of the surrounding atmosphere, then the lower temperature is a condition of the deposit of dew. Take another example. Brewster took impressions from a piece of mother-of-pearl in a cement of resin and beeswax, in balsam, in fusible metal, in lead, in gumarabic, in isinglass, etc. In all cases the same iridescent colour appeared. But the only character which these substances had in common was the form of the surface produced by the impression of the piece of mother-of-pearl. Hence that form of surface must be a condition of the iridescent colour.

No scientific, or other, method is fool-proof, and

the Method of Agreement is perhaps least so. In the absence of an adequate analysis of circumstances, even in spite of the most scrupulous caution, mistakes are easily committed. One might conceivably argue that since different drugs, which had proved fatal, appeared to have nothing in common except water, or moisture, therefore water is poisonous. But the absurdity of this is too transparent. It is different when the common antecedent is something complex, and insufficiently analysed and understood at first. For example, at one time the presence of marshes was regarded as the common condition of epidemics of malaria. This was really the clue to the subsequent discovery that mosquitoes (which abound in marshy regions) are the carriers of malaria. For the rest, the chief dangers to which the Method of Agreement is exposed are: (*a*) that a relevant circumstance may be overlooked; (*b*) that consequents not precisely alike, except perhaps for practical purposes, may be treated as essentially similar, and assigned to a common circumstance, which is not the condition at all (for instance, the above example relating to drugs); (*c*) that the antecedent and consequent may both be the consequents of the same condition, or set of conditions, as is the case, for example, with the sequence of day and night, or the sequence of the phases of the moon, or the sequence of the seasons of the year—day is not a condition of night, or *vice versa*, but the whole sequence of day and night is conditioned by the rotation of the earth, and similarly with the other sequences just mentioned.

§ 6. *The Method of Residues.*

If part of a complex result can be accounted for by certain antecedents which are known to have been operative, and the nature of whose consequents is already known as the result of previous investigations, then the residue of the complex result must be due to the remaining operative condition or conditions. Symbolically :—

$$\underbrace{a\ b\ c\ d\ \ldots}_{w\ x\ y\ z\ \ldots};\quad \left[\underbrace{a\ b\ c\ \ldots}_{w\ x\ y\ \ldots}\right];\quad \text{therefore}\quad \begin{array}{c} d\\ |\\ z\end{array}.$$

Sometimes such other conditions are already known to be present and it is only a matter of determining their precise effects. More often their presence is not even suspected until the residual phenomena compel one to search for them. For example, the weight of coal in a truck may be determined as a residual phenomenon if one knows the net weight of the empty truck and deducts it from the gross weight of the loaded truck. Or the resistance of the air on the trajectory of a bullet may be determined by observing the deviation of its actual trajectory from that which could be accounted for by the value and direction of the propelling force and the force of gravitation. In both these examples the presence of the residual condition is known, only its weight, or influence, has to be determined. In other cases, and these are the most important cases, it is different. Thus, for example, the existence of argon was not suspected until the residual density of atmospheric nitrogen

(that is, nitrogen obtained from the atmosphere by removing impurities, moisture, oxygen, etc.) in comparison with chemical nitrogen (that is, nitrogen prepared from nitrous oxide, or nitric oxide, etc.) was observed. Similarly the existence of the planet Neptune was not thought of until the residual deviation in the orbit of Uranus made astronomers look for it. Nor was the velocity of light known, or surmised, until attention was directed to it by the (residual) difference in the observed periods between successive eclipses of Jupiter's satellites, according as the earth in its orbital motion was moving towards or away from Jupiter.

§ 7. *The Joint Method of Agreement and Difference.*

If a group of several instances in which a phenomenon occurs have nothing relevant in common except a certain antecedent, while another group of similar instances in which the phenomenon does not occur have nothing relevant in common except the absence of that antecedent, then that antecedent is a condition of that phenomenon. Symbolically :—

Positive group : $\underbrace{a\,b\,c\,d\ldots}_{w\,x\,y\,z\ldots}$; $\underbrace{b\,d\,f\,g\ldots}_{x\,z\,s\,t\ldots}$; $\underbrace{d\,f\,k\,l\ldots}_{z\,s\,p\,r\ldots}$;

Negative group : $\underbrace{b\,c\,f\ldots}_{x\,y\,s\ldots}$; $\underbrace{b\,g\,k\ldots}_{x\,t\,p\ldots}$; $\underbrace{c\,l\,a\ldots}_{y\,r\,w\ldots}$;

therefore $\begin{matrix} d \\ | \\ z \end{matrix}$.

For example, Lord Avebury found that various insects differing in many respects but all having

compound eyes can see at a distance, whereas other insects having ocelli, but not compound eyes, cannot see at a distance. He concluded, accordingly, that compound eyes are a condition of distant vision. Another example, Darwin observed that many plots of land containing all of them plenty of earth-worms, although otherwise very different in character, became covered increasingly with vegetable mould, whereas, on the other hand, many plots of land not essentially unlike the former plots as a whole, but deficient in earth-worms, did not get covered with vegetable mould. He therefore concluded that the vegetable mould is due to the agency of earth-worms.

It should be observed that the positive and negative instances which are sufficient for the application of the Joint Method are not such as would make it possible to employ the Method of Difference. There may be no negative instance sufficiently similar to any of the positive instances to meet the requirements of the latter method. All that can be said is that if the whole group of positive instances is regarded as though it were one positive instance, and if the whole group of negative instances is regarded as though it constituted one negative instance, then the Joint Method appears as an approximation to the group-form of the Method of Difference.

§ 8. *Relevance.*

The application of the simpler inductive methods, and, for the matter of that, even of other methods,

is only possible in so far as the vast majority of antecedents or accompaniments of the phenomenon investigated can be ignored as irrelevant to the study of the phenomenon in question. Certain philosophers who took Mill's formulations of the canons of induction too literally, and rather unintelligently, had no difficulty in pointing out the impossibility of the application of these canons to the actual phenomena of our infinitely complex world. And in our formulation of the inductive methods the word *relevant* plays an important part. The question naturally arises as to how it can be known whether any antecedent or accompaniment is relevant or irrelevant.

The answer is that it is impossible to indicate any definite and reliable mark of relevance or of irrelevance among the antecedents or accompaniments of any phenomenon that may be investigated. The fact is that in every kind of investigation common sense, previous knowledge of kindred phenomena, and some originality, and spirit of adventure are indispensable. The study of logic and scientific method is a supplement to, not a substitute for, common sense, to say nothing about the other requisites indicated. Unfortunately there are lots of people, even writers, whose sense of relevance is not well developed. Not only the daily press but lots of books on philosophy, and even on logic, betray this defect. All one can do in a short space is just to indicate briefly how investigators generally proceed in discriminating between what is relevant and what is irrelevant to their investigation

The most important clue is previous knowledge. Those antecedents and accompaniments whose effects are already known, and are known to be different from the phenomenon under investigation, are usually dismissed as irrelevant, unless there is some *primâ facie* ground for regarding them as influencing the phenomenon under investigation in some measure by way of resistance or of modification. Our knowledge of what is relevant, like all other human knowledge, can only be developed and confirmed by more knowledge.

Another factor involved is almost too nebulous or vague for precise description, yet, as every investigator knows, it is a very real influence. I mean just the vague feeling or intuition that certain things are relevant, while others are not. This feeling "in our marrow," to use a fairly familiar phrase, is probably itself to a large extent the outcome of previous experience which has not yet unfolded as explicit thought. Unfortunately it is also apt to be wrong sometimes. But so it is, and the wise man makes the best of things, and wastes no tears over the absence of sure signs, or of fool-proof methods.

CHAPTER XX

THE STATISTICAL METHOD

§ 1. *The Method of Simple Enumeration and Exact Enumeration.*

The Inductive Methods described in the preceding chapter can only be applied fruitfully where the facts investigated can be analysed adequately, and examined under sufficiently varied conditions. This is equally true of the more advanced methods which will be considered later. Now these requirements cannot always be satisfied. The facts investigated may be too complicated for adequate analysis, and may not be observable under sufficiently varied circumstances. In such cases it is impossible to ascertain with confidence the thread of connection between conditions and consequents by means of the above-mentioned inductive methods. The kind of facts here considered may be indicated by reference to meteorological, economic, social, medical and various biological problems.

Popular thought, impelled by practical needs and by its proverbial incapacity to suspend judgment, resorts in such cases to what is known as the Method of Simple Enumeration. That is to say, the concurrence or sequence of certain attributes, or circumstances, or events is noted, and, if a concurrence or sequence is observed a considerable number of times, it will be assumed that the facts or events in question are connected with one another as condition and consequent, or, to use a more

popular mode of expression, as cause and effect. Popularly this method, such as it is, is employed even in cases where more satisfactory methods might be applied. No attempt is usually made to observe sufficiently varied instances of concurrence or of sequence (as is required, for example, for the application of the Method of Agreement), nor, as a rule, is attention paid to exceptions. It is only called a method by courtesy. It is a loose habit of mind rather than a scientific method. Its uselessness is attested by many popular fallacies and superstitions. A full moon is commonly believed to bring fine weather, because the two have often been observed together; it is, of course, on fine nights only that the average person takes cognisance of the full moon, and he does not think of ascertaining if the weather is not also bad sometimes when there is a full moon. Similarly, many otherwise intelligent people still stand in awe of the number 13, which private hosts, as well as hotel managers, carefully avoid.

Now, the scientific method, which is usually employed in such complex cases as are not amenable to the other inductive methods, is the Statistical Method. We have already made our first, though slight, acquaintance with this method as an auxiliary to the Method of Classification, namely, as an aid to adequate description in certain types of cases (see Chapter XVII, § 2). It may be used similarly as a descriptive auxiliary to the Evolutionary Method. Indeed, much of the material examined in connection with problems of biological

evolution has been, and is being, classified and tabulated in accordance with Statistical Methods. But the most important use of the Statistical Method is as an independent scientific method for ascertaining connections, or laws, and regularities. Like the Method of Simple Enumeration, it notes concurrences; but, unlike it, it is careful also to note and record exceptions, to make observations over as large and varied a field as possible, and then to proceed cautiously to interpret the whole of the observations made and recorded.

§ 2. *Statistical Processes.*

Scientific investigation is always concerned with the discovery of the relationship between two or more attributes or variables. By an "attribute" is here meant anything the bare presence or absence of which can be noted and counted, but which is not otherwise measurable; by a "variable" is here meant anything that has a magnitude that is measurable, and which may be present in different magnitudes. The Statistical Method seeks to discover whatever regularity might subsist between two or more attributes, or two or more variables. Now the concurrence of two or more attributes, or the correspondence of two or more variables, may be merely a chance coincidence, or it may be the result of some direct or indirect connection between them. By observing only one instance, or a small number of instances, of the concurrence or the correspondence, and that under conditions beyond our control, and under circumstances not adequately

known, it may be impossible to distinguish between a casual and a causal concurrence or correspondence. But the observation of a large number of instances taken from a wide range, and an exact enumeration of both positive and negative cases, and of variations between series of cases, may enable us to draw a highly probable conclusion about the connection between the phenomena in question. Such procedure, based on exact enumeration, is of the essence of the Statistical Method.

The processes or stages involved in a complete application of the Statistical Method may be described as follows :—

(*a*) *Collection of Material.* The facts or data under investigation, or, more usually, adequate samples of them, are observed, counted or measured, and described in a way relevant to the problem in hand. The measurements and descriptions must obviously be sufficiently accurate, if they are to be of value, but the degree of precision required will vary with different investigations. To avoid one-sidedness, it is desirable that the facts should be collected from as wide and varied a field as possible. In some statistical inquiries the data are collected by means of questionnaires, sometimes of an official character (like the census, or various trade schedules), sometimes of a private character (like those sent out by the late Sir Francis Galton, or by Professor Karl Pearson, and others). In such cases it is necessary, though not always easy, so to frame the questions as to reduce to a minimum the danger of obtaining misleading answers.

(*b*) *Classification, Tabulation and Correlation of Material.* The facts or data are then classified and tabulated with respect to certain attributes, or variables, in which the investigator is interested. Examples of simple Classification and Tabulation have already been given in Chapter XVII, § 2, and may be referred to now. For merely descriptive purposes, such simple tables dealing only with one attribute, or one variable, may be useful; but for further investigation we require tables giving two or more attributes (such as, say, the colour of eyes and of hair), or two or more variables (such as the supply and the price of wheat, or of some other commodity, over a period of years, or the temperature of different geographical areas and their latitude, or the length and breadth of leaves). The following table may serve as an example of a simple type of correlation or contingency table, as such a two-fold (or multiple) table is called.

Eye colour.	Hair Colour.		Total.
	Fair.	Dark.	
Light ..	2,714	3,129	5,843
Brown ..	115	726	841
Total ..	2,829	3,855	6,684

Proportion of light-eyed with fair hair $\frac{2,714}{5,843}$ = 46 per cent.

Proportion of brown-eyed with fair hair $\frac{115}{841}$ = 14 per cent.

(*c*) *Summarizing the Tables.* The data classified and tabulated are often very numerous and complicated. It may be difficult to see the wood for the trees. A concise summary of the results by the aid of averages, coefficients of association and of correlation is, therefore, helpful or even necessary. Graphs and other diagrams are also a useful aid for bringing the results home. It is this stage especially that calls for a knowledge of mathematics and of statistical technique. (See Chapter XVII, § 2, above.)

(*d*) *Critical Interpretation.* As a result of the foregoing processes it may next be possible to state the extent and character of the relation between two or more attributes or variables that have been investigated. There may be no association or correlation [1] between them at all. That is to say, the occurrence of the one attribute, or the value of the one variable, may show no regular correspondence with that of the other. They may occur together or correspond sometimes, but that may be a mere coincidence. On the other hand, the presence or absence of the two attributes, or the variations of the two variables, may show a regular correspondence. In that case they may be intimately connected; their concurrence or correlation may be something more than a mere coincidence. This may be the case also even when the association

[1] The term "association" is usually employed to express the relation between two or more attributes, as defined above. The term "correlation" is, on the other hand, usually restricted to express relation between variables. But the distinction is not always adhered to.

of the two attributes, or the correlation of the two variables, is not complete but only partial. In the best cases we are thus led to the discovery of a law, that is to say, a relation which we have reason to believe to be uniform. In somewhat less successful cases we may still be able to formulate a regularity of some kind.

The establishment of some law or regularity of connection may be said to be the natural end of the Statistical Method at its best. The word "end" is here used with conscious ambiguity. The Statistical Method, like all scientific methods, aims at the discovery of general truths, if possible. On the other hand, as soon as such general truths are discovered in any department of inquiry, the Statistical Method is apt to be superseded. There is no further interest in noting the frequency of the occurrences or facts in question when their laws are already known. For example, there was a time in the history of Astronomy when records were kept of solar and lunar eclipses. It was on the strength of certain observed, but as yet unintelligible, cycles that the ancients already foretold eclipses with some accuracy. But since the laws of the occurrence of eclipses have been discovered there is no further need to keep statistical records of their occurrence. Eclipses can be foretold with great accuracy and certainty.

Laws, however, are not so easily discovered, and the Statistical Method is not fool-proof. Great care is required for the correct interpretation of associations and correlations. Even a high degree of

association of attributes, or of correlation of variables, may be no conclusive evidence of real connection. A few simple examples may make clear some of the types of rash interpretation. The fact that a high percentage of full-moon nights are fine is no evidence of real connection between full moon and fine weather. A comparison with other nights, when there is no full moon, shows that the percentage of fine nights when the moon is not full is just as high. Again, the fact that the mortality of babies who use comforters is six times as great as that among children who go without comforters, is, according to Professor Pearson, also no evidence of a connection between the use of comforters and infant mortality. The higher mortality may be due to hereditary weakness of the children who use comforters; the very use of comforters may only be a symptom of the children's weakness and consequent irritableness. Apparently, with a little ingenuity it is possible to correlate the spread of cancer with the increased importation of apples, and the expenditure on the Navy with the growing consumption of bananas, at least so Professor Pearson suggests. If so, it is obvious that mere statistical technique is no adequate substitute for common sense, and scientific insight.

§ 3. *Kinds of Association and Correlation.*

In its most fruitful applications the results of the Statistical Method are in some ways very like those of the Method of Agreement and the Method of Concomitant Variations, although the processes are

different in some respects. In any case, the associations or correlations established by the Statistical Method, and the concomitant variations shown by the Method of Concomitant Variations, exhibit analogous types. There are Positive and Negative Associations and Correlations, just as there are Direct and Inverse Concomitant Variations; and there are Simple and Complex Associations and Correlations, just as there are Simple and Complex Concomitant Variations. (See Chapter XIX, § 4, above.) One important difference is noteworthy: whereas the Method of Concomitant Variations, like the other inductive methods, is only concerned with the discovery of uniform relations, or laws, in the stricter sense, the Statistical Method is concerned with the discovery of partial associations and correlations, as well as with the discovery of complete associations and correlations. Moreover, for purposes of merely qualitative induction (see Chapter XIX, § 4), the Method of Concomitant Variations can be employed in cases in which quantitative variations can be observed but cannot be measured with any accuracy; the Statistical Method, on the other hand, is only applicable to phenomena which, directly or indirectly, can be measured with accuracy.

The association between two attributes is said to be positive if the presence of one is accompanied by the presence of the other; it is said to be negative if when one of them is present the other is absent. Similarly, the correlation of two variables is said to be positive if an increasing value of one of them

corresponds to an increasing value of the other. It is said to be negative when an increasing value of the one corresponds to a diminishing value of the other. Negative association or correlation must be distinguished from the absence of association or of correlation. There is an absence of association between two attributes when the presence or absence of one of them corresponds in no way to the presence or absence of the other, their concurrence or otherwise being a matter of chance. Similarly, the absence of correlation between two variables means the absence of any kind of correspondence between their values. In contrast with such absence of correlation and association, negative correlation and negative association are real correlation and real association, just as, in the case of concomitant variation, inverse concomitant variation is also a real form of concomitant variation, and quite different from an absence of concomitant variation.

Again, the association between two attributes, or the correlation between variables, may be complete, so that we can express it in the form of a general truth or law, such as "all cases of A are cases of B," or "$A = c(B)$," where c stands for some ascertained constant. Complete association or correlation is sometimes expressed by 1 and sometimes (in the U.S.A.) by 100; and positive and negative association or correlation are expressed by $+$ and $-$ respectively. So that $+1$ (or $+100$) would express complete positive association or correlation, and -1 (or -100) would express

complete negative association or correlation. Absence of association or correlation is expressed by 0. Partial association or correlation will be expressed by numbers intermediate between 0 and 1 (or 100). Such numbers are known as coefficients of association (sometimes symbolized by Q), or coefficients of correlation (usually symbolized by r). The coefficient $+ \cdot 8$ (or $+ 80$) would be considered to express a high degree of positive connection. It is in fact the coefficient of correlation between the stature of man and his cubit (that is the length of the arm from the elbow to the tip of the middle finger), also between the cubit and the height of the knee.

§ 4. *The Value of Descriptive Statistics.*

The results of the application of Statistical Methods are often valuable, even when they do not lead to explanation, or when they do not establish any connection between the phenomena, or even disprove an alleged connection. Through exact descriptions, by means of accurate counting and measuring, classifying and tabulating, the phenomena under investigation assume an orderliness which renders them easier to grasp, and such orderliness clearly paves the way for future discoveries and explanations. Many of the results of the application of Statistical Methods are also very useful to the individual and to society. This is evident from the fact that the whole business of insurance rests on statistical calculations and processes. Life contingencies, or mortality tables,

are an excellent help to insurance companies, even if they throw no light on the complex conditions which determine life and death. The statistics of births, marriages, imports, exports, etc., furnish a certain amount of guidance in practical affairs, besides preparing the ground for further sociological and economic research. Even the knowledge of the number of suicides per thousand of the population in a given country for a given period of years may throw some light on social and economic conditions. When the rate of such an occurrence shows comparative regularity for a period of years, that may be taken as an indication that certain social or other conditions have not changed much during that period. But that of itself is no evidence that the relevant conditions, whatever they may be, cannot or will not change; and, with any change in these conditions that may come, the rate of occurrence in question is also liable to change. Within certain limits, or with reference to a short future period, it may be safe enough to rely upon the regularity of past rates, so long as there is no evidence of any striking changes in the relevant conditions. But it is an unscientific extravagance to raise any such observed regularity of the past to the rank of an invariable and inviolable law. Yet that is what Buckle did, when he maintained that in a given state of society 250 persons *must* in the course of each year put an end to their lives; that so and so many letters *must* miscarry, and so on. It happens only too frequently that people fail to realize the nature of their assumptions and inferences, or, in-

deed, fail to realize that they are making assumptions and inferences at all, when they treat the chronicle of the past as an almanac for the future.

While there can be no two opinions about the helpfulness of statistical technique for the orderly description and preparation of material for further scientific investigation, there is no such confident unanimity about the self-sufficiency of statistics as an independent method of scientific interpretation. This is intelligible. The correct interpretation of phenomena requires, above everything else, a thorough familiarity with the phenomena themselves. An expert chemist will achieve far more with inferior apparatus than an amateur can hope to achieve with the best apparatus. Similarly, an expert biologist or psychologist is likely to interpret his facts more accurately, even if he is not an expert in statistical technique, than a statistical expert who is an amateur biologist or psychologist. By setting up statistics, not only as an independent method, but as an independent science, a certain amount of encouragement may be given unintentionally to the conceit that anything which can somehow be counted and measured is grist for the statistical mill, and can be manipulated and interpreted adequately by any statistician. But statisticians themselves, at least when commenting on the work of their colleagues, have had frequent occasion to point out the inadequacy of statistical technique alone to the complete solution of scientific problems.

CHAPTER XXI

THE DEDUCTIVE-INDUCTIVE METHOD

§ 1. *The Combination of Deduction and Induction.*

In science, as in every kind of study, knowledge already acquired facilitates the acquisition of further knowledge. This was illustrated, to some extent, in connection with the Method of Residues (Chapter XIX, § 6). Considerable progress in the development of science may be rendered possible by combining the simpler inductive methods with deductive reasoning, either of them being used to confirm or to extend the knowledge obtained or obtainable by the other. The combination of deduction with induction has been named, by John Stuart Mill, the "Deductive Method"; but as this is rather liable to be confused with mere deduction, which is only one constituent of the combined method, it may be better to describe it as the Deductive-Inductive Method. As applied to the study of natural phenomena, Mill distinguishes two principal forms of the Deductive-Inductive Method, namely: (*a*) that in which the deduction precedes the induction, and (*b*) that in which the induction precedes the deduction. The former he calls the "Physical Method"; the latter he calls the "Historical Method." The distinction is of no fundamental importance, and the names are very inappropriate. Both forms of the Deductive-Inductive Method are employed in Physics, and in other sciences. More important than the order in which the two parts of the method

are applied, is the nature of the circumstances leading to the application of this whole method. Briefly, there are three kinds of occasions on which the Deductive-Inductive Method is employed: (1) When an hypothesis cannot be put to the test directly, but only indirectly; (2) when the attempt is made to systematize already accepted inductions or laws under more comprehensive laws or theories; (3) when, owing to the difficulties of the problem, or to the lack of sufficient and suitable instances of the phenomena studied, deduction and induction are employed by way of mutual support. Each of these types of cases may now be considered separately.

§ 2. *The Indirect Verification of Hypotheses.*

Sometimes an hypothesis, stating the possible nature of the connection between the phenomena studied, cannot be put to the test directly; only its consequences can be tested by observation or experiment in the light of already established knowledge (frequently including mathematical knowledge). The implications of the hypothesis are then deduced or calculated, by the aid of mathematical or other forms of deductive reasoning, until we arrive at such consequences as can be put to the test of observation or experiment. A simple example may help to make the difference clear. For this purpose, two hypotheses of Galilei will serve. At the time of Galilei there was still current the Aristotelian hypothesis that the velocity of falling bodies varied with their weight. Galilei opposed this view and put forward the hypothesis

that all bodies, no matter what their weight may be, fall through the same distance in approximately the same time (allowing for the resistance of the air). These conflicting hypotheses could be tested directly by dropping simultaneously bodies of different weights from the same height. Galilei did test them in this way, by dropping bodies of different weights from the leaning tower of Pisa, and thereby he disproved the Aristotelian hypothesis, while confirming his own. The method employed was that of Concomitant Variations, only the result was negative—the difference, or variation, in weight was not followed by any difference in velocity. Such simpler inductive methods, however, did not suffice when Galilei next undertook the task of ascertaining the real law of the velocity of falling bodies. After trying various hypotheses, there occurred to him, eventually, the hypothesis that a body starting from rest might fall with uniform acceleration, and that its velocity might vary with the time of the fall. But he could not think of any method of testing that hypothesis directly. By mathematical deduction, however, he concluded that if a body did fall in the way suggested by his hypothesis, then the distance through which it would fall should be proportionate to the square of the time of its fall. This consequence of the hypothesis could be tested directly, by comparing the actual distances traversed by falling bodies during different times, or by comparing the times taken by the fall through different distances.

As another example of this use of the Deductive-

Inductive Method we may refer to Newton's hypothesis that the orbital movement of the moon is determined by terrestrial gravitation. This hypothesis could not be tested directly by any of the simpler methods of induction alone. But, by deductive reasoning and calculation, Newton arrived at the conclusion that, if his hypothesis were true, then the moon should be deflected from its rectilinear path at the rate of approximately 16 feet per minute. Now, this consequence of the hypothesis could be tested, by observing and determining the orbit and period of the moon. Eventually the hypothesis was actually confirmed in that way, though, owing to a misconception about the length of the earth's radius (which was one of the data of his calculation), Newton abandoned his hypothesis for a long time. Still other instances of this use of the Deductive-Inductive Method are the Undulatory Theories of Light and of Sound.

§ 3. *The Systematization of Laws.*

The more developed sciences constantly endeavour to link up systematically such laws or regularities as they have already discovered. The greater the knowledge already possessed the more possible is it usually to interconnect it into a coherent system. Conversely, the more systematic the knowledge becomes, the deeper and more coherent does our insight into the phenomena become. Speaking metaphorically, each isolated law reveals a continuous thread in the tangled fabric of Nature, but still only a single thread; when various laws are

found to be systematically interrelated, then they reveal something of a whole pattern in the fabric of Nature. The usual way of establishing such systematization is by discovering some hypothesis from which certain laws, already obtained by previous inductions, but apparently standing in no relation to one another, can all be derived by deductive reasoning. The Law of Universal Gravitation is a familiar example of this kind of procedure. Kepler had discovered three most important laws of planetary motion, by induction from numerous astronomical observations made by Tycho Brahe and himself. The three laws were: (*a*) that the planets move in elliptic orbits having the sun for one of their foci; (*b*) that the velocity of a planet is such that an imaginary line (called the radius vector) joining the moving planet to the sun sweeps out equal areas in equal intervals of time; and (*c*) that the squares of the times which any two planets take to complete their revolutions round the sun are proportional to the cubes of their mean distances from the sun. Newton showed that these laws could all be deduced from the law that the planets (or, more generally still, all particles of matter) tend to move towards each other with a force varying directly as the product of their masses, and inversely as the square of the distances between them. In that way, laws (and phenomena) which appeared to have nothing to do with one another were shown to be expressions (or manifestations) of the same systematizing principle.

Additional instances of this use of the Deductive-

Inductive Method are furnished by the Kinetic Theory of Gases in its relation to Boyle's Law, to the Law of Charles, to the Law of Avogadro, etc., and by the Undulatory Theory of Light in relation to Snell's Law of Refraction, etc.

§ 4. *The Mutual Support of Deduction and Induction.*

In the study of certain kinds of highly complex phenomena which are beyond the control of the investigator, such as economic and other social phenomena, it is very unsafe to put much faith in the necessarily inadequate applications of the simpler inductive methods, and even more unsafe to trust purely deductive reasoning from a few elementary laws of human nature, etc. It is not safe to trust purely deductive reasoning, because there is the risk of overlooking all sorts of modifying or counteracting factors, so that the concrete result may be very different from that anticipated on deductive grounds. And, in the kind of cases here contemplated, it is also unsafe to put implicit trust in the inductions alone, because they are based on a comparatively few instances observed with difficulty under circumstances which are extremely complicated, not varied in the way required for the cogent application of the simpler methods of induction, and altogether beyond the control of the investigator. In such cases one does the best he can by the aid of both deduction and induction, and if the two modes of procedure converge towards the same conclusion, then one's confidence in the result is naturally greater.

An interesting example of this use of the Deductive-Inductive Method is contained in Herbert Spencer's *Principles of Sociology*. The following is a very bald summary of the argument, giving just enough to bring out the character of its method, and no more. Spencer's aim is to prove a connection between industrialism and free institutions, or, conversely, between militarism and lack of freedom. The first part of the argument (*a*) is inductive, involving rough applications of the Methods of Agreement, Difference, and the Joint Method. But, as the instances he can draw upon are rather few for such a complex problem, and not related to one another in precisely the way required for a rigorous application of the inductive methods, he endeavours to confirm his inductive conclusion by (*b*) independent deductive reasoning from the nature of the case, in the light of what is known of human nature.

(*a*) In Athens, where industry was regarded with comparative respect, there grew up an industrial organization which distinguished the Athenian society from adjacent societies, while it was also distinguished from them by the democratic institutions that simultaneously developed. Turning to later times, the relation between a social regime predominantly industrial and a less coercive form of rule than is usually found in societies which are predominantly militant is shown by the Hanse towns, by the towns of the Low Countries out of which the Dutch Republic arose, by Norway, by the United States, by Britain, and the British colonies. Along with wars less frequent, and along with an

accompanying growth of agriculture, manufacture, and commerce, beyond that of continental states more military in habit, there has gone in England a development of free institutions. As further implying that the two are related as cause and consequence, there may be noted the fact that the regions whence changes towards greater political liberty have come are the leading industrial regions, and that rural districts, less characterized by constant trading transactions, have retained longer the earlier (militant) type with its sentiments and ideas.

(*b*) The pervading traits in which the industrial type differs so widely from the militant type, originate in those relations of individuals implied by industrial activities, which are wholly unlike those implied by militant activities. All trading transactions are effected by free exchange. For some benefit which A's business enables him to give, B willingly yields up an equivalent benefit. This relation in which the mutual rendering of services is unforced and neither individual is subordinated, becomes the predominant relation throughout society in proportion as the industrial activities predominate. Daily determining the thoughts and sentiments, daily disciplining all in asserting their own claims while forcing them to recognize the correlative claims of others, it produces social units whose mental structures and habits mould social arrangements into corresponding forms. There results a type of society characterized throughout by the same individual freedom which every commercial transaction implies. In the militant type,

on the other hand, the nation is essentially an army sometimes mobilized, at other times quiescent. And as the soldier's will is so suspended that he becomes a mere instrument of his officer's will, so the citizen of a militant regime is overruled by the government.

If inductive inference sometimes needs support from deduction, purely deductive reasoning (except perhaps in pure mathematics) stands in even greater need of inductive confirmation, especially in the case of complex phenomena. The history of science, especially of economic and social science, can point to many cases which should serve as a warning in this respect. Ricardo, for instance, arguing deductively, maintained that the continuous increase in population would necessitate the cultivation of less and less fertile soils; this would raise land rents, and increase the price of food. This deductive conclusion was falsified by improvements in agricultural methods, and by the cultivation of fertile soils at great distances, which was rendered possible by developments in transport. Similarly, some of the Malthusians, relying on inadequate deductions, arrived at rather pessimistic conclusions about the future of the working classes. It was argued that any improvement in the regularity and amount of their wages would only encourage them to have still larger families, whose additional needs would continue to keep them at the poverty line. But the subsequent investigations of Charles Booth showed clearly that, as a matter of fact, the families of the working classes steadily diminished in numbers,

and their standard of life steadily became higher, as their income improved in amount and in regularity. Deductive reasoning is, of course, sound enough as far as it goes; there is nothing intrinsically wrong with it. But, as already remarked, it is liable to be too abstract, in the sense of not taking into account all the factors involved. And it is just this that gives what little justification there is for the hackneyed dictum of men of practical affairs, namely, that this or that may be all right in *theory*, but will not work in *practice*. Except in purely hypothetical cases, what is true in theory is meant to be true in practice. But deductive theory is liable to overlook factors, whose actual influence is in no way diminished by being forgotten.

§ 5. *The Value of the Deductive-Inductive Method.*

Contrary to what might, at first, be expected, the indirect, more roundabout method of verifying hypotheses and establishing connections between phenomena, is scientifically more valuable than the direct method. The simpler methods of induction are frequently applicable where, as yet, there has been no great development of the science concerned. They are applicable where comparatively little is known as yet; but the Deductive-Inductive Method, especially in its more complicated forms, demands a considerable amount of systematic knowledge, and so presumes a systematization on the part of the science in question, which, in its turn, it helps to systematize still further. For example, the ancient astronomers (long before Thales) had noticed that

solar and lunar eclipses occurred in cycles of 6,581 days. If, on the strength of such a purely empirical periodicity, they conjectured when the next solar or lunar eclipse would occur, then they could verify their conjecture or hypothesis directly by waiting till the proper time. This kind of conjecture required comparatively little previous knowledge, and its verification added very little to the existing stock of knowledge, beyond confirming slightly the probable correctness of the assumed periodicity. On the other hand, the modern astronomer dealing, say, with the lunar theory, has a much more complicated task before him. He puts to a severe test all the great astronomical ideas already accepted, and their systematic coherence, which he strengthens or improves by the very use to which he puts them in the long chain of deductions by which he arrives at conclusions that can be tested by actual observation. He must know the approximate masses and the relative positions of the planets of the Solar System; and has to rely on the accuracy of the Law of Universal Gravitation. He must calculate the constant changes in the positions of the earth while the moon is moving round it. And so on. This involves the most elaborate deductive calculations by the aid of differential equations. But when the anticipated positions of the moon, inferred by the aid of these calculations, are approximately verified, then the whole group of ideas involved is thereby cemented into a coherent system or a "theory," as such an inter-connected system is usually called. Similarly with other theories.

CHAPTER XXII

PROBABILITY

§ 1. *The General Nature of Probability.*

Probability is usually contrasted with certainty, and both terms apply only either to judgments or to propositions (which are the verbal expressions of judgments). There are cases in which we do not feel competent to judge at all, when we simply suspend judgment, and then the question of probability or certainty does not arise. But when we do judge, then the judgment is entertained either with relatively complete confidence, which is called certainty, or with some lesser degree of confidence. It is this lesser degree of confidence, the confidence that falls short of certainty, that is usually designated as probability. It is more convenient, however, to regard complete confidence or certainty as the limiting case of probability. We then have a continuous series or scale of probability varying from the lowest to the highest degree of confidence.

To avoid possible misunderstandings several things must be borne in mind. Certainty or uncertainty may originate in different ways. It may be the result of the moods and dispositions, hopes and fears, habits and prejudices of the individual who is judging. These are subjective, personal factors, which vary from individual to individual. Some people confidently expect a certain event merely because they wish it to happen, and they are sanguine by temperament. Others may be extremely

uncertain about some event, either because they are not keen about it, or because they have a morbid habit of expecting the fates always to thwart their wishes. In contrast with such merely subjective causes of certainty or uncertainty, there are objective or logical grounds on which they may be based. They are the kind of grounds to which we usually appeal when we try to convert others to our views, and do not rely entirely on our powers of bullying or of coaxing. Such rational grounds do not vary from individual to individual, but are valid for all intelligent beings. Now, the kind of uncertainty with which probability is concerned is that based on rational grounds. It is not concerned with mere feelings of conviction arising we know not how, but with those varying degrees of assertiveness which are correlated with corresponding degrees of rational support which our judgments find in the available evidence.

Hence, probability may be said to be concerned with the problem of *rational* belief. Many of our judgments we are quite sure of, without any inspiration from our feelings or prejudices; sometimes indeed, in spite of them. "The square on the hypotenuse of a right-angled triangle, is equal to the sum of the squares on the other sides," however much one may dislike the Pythagorean theorem. On the other hand, there are many things about which we cannot judge with certainty. We may have some grounds for thinking that "*S* is *P*," but the grounds may not be conclusive. We must then content ourselves with the judgment, "*S* may be *P*," or

"*S* is probably *P*." It may be as well to emphasize at once the fact that probability and certainty refer to judgments about things or events, not to things or events themselves. Language, in its commendable aim at brevity, is rather misleading sometimes. Things just are what they are, and events just happen as they do. It is only our judgments about them that can be either probable or certain. It would indicate no additional character in the thing or in the event referred to in the statement, "snow is white," or "the water is freezing," if one were to insert the word "certainly" or "probably" after the word "is." The insertion of either of these words would only indicate a difference in the degree of our confidence in our own judgments, nothing else. As a matter of convenience, however, one may and does speak of "the probability of events," when what one really means is "the probability of the judgment that the events will happen"—much in the same way as one continues to speak of "sunrise" and "sunset," etc., and would consider it tiresome pedantry to have to express himself accurately in accordance with the heliocentric astronomy.

Again, propositions which are the result of direct observation are normally entertained with certainty. It is only when we are observing under difficulties that we become uncertain, and then the uncertainty attaches not so much to the sense-impressions as to the elements of interpretation which enter into the complex whole of our perception. Generally speaking, therefore, it may be said that the question of

probability arises chiefly in connection with inferred judgments, that is, judgments which rest on evidence; and degrees of probability may then be said to be correlated with degrees of evidence, or degrees of cogency in the evidence. Accordingly, there are as many degrees of probability as there are kinds of available evidence. Moreover, the probability of our judgment relating to the same things may vary considerably from time to time, as more and more evidence comes to light. Our increasing knowledge of the evidence may, of course, have no influence on the thing or event in question; but it is all-important in determining the rational justification of our judgments relating to it.

In most of the affairs of life we have to come to a decision on evidence which is not conclusive, so that our judgments are not certain, only more or less probable. "Probability," as Bishop Butler has said, "is the very guide of life." This very fact has imparted a special interest to the study of probability and the problem of its accurate measurement. Indeed, the mathematical treatment of probability is concerned almost exclusively with the measurements of probability. But not all cases of probability are really measurable, not even all those cases in which differences of degree are readily distinguishable. For example, in a law-court one may feel justified in believing it to be more likely that witness A had told the truth rather than witness B, and yet one may be unable to assign a definite truth-probability to the statement of either. Or a witness may think it more likely that a certain

event had happened at one time than at another, and yet he may be unable to estimate the probability of either. No doubt it is very trying to have to depend on such vague estimates as "rather probable," "quite probable," "very probable," etc., especially in the course of trials on which issues of life and death may depend. One can understand, accordingly, the motive behind Bentham's suggestion for the use of a probability-scale on which witnesses and judges might indicate the degrees of certainty (from 0 to 10) of their evidence or conclusions. But it is difficult to see the practicability of the suggestion. With unconscious humour the proposed probability-scale has been called a probability-*thermometer*. If adopted, it would probably serve to indicate more often warmth of feeling than the dry light of reason.

There are cases, however, in which probability may be measured with great precision. It is these cases that have chiefly interested writers on Probability, as well as those ardent wooers of Fortune who seek to reach her by short cuts. These measurable cases are of two principal types, namely, those which can be calculated *a priori*, and those which can only be calculated *a posteriori*. By the *a priori* cases we mean those which can be determined by reasoning from the nature of the case, independently of actual observations of the kind of events contemplated. By the *a posteriori* cases we mean those which can only be determined by the aid of such observations.

§ 2. *The Deductive Calculation of Probability.*

The *a priori* (or deductive) method of calculating probabilities is possible on the following conditions: (1) We must know the total number of mutually exclusive alternatives, one or other of which must happen. (2) These alternatives must be equally likely. (3) We must know how many of the alternatives are favourable to the event concerned. The probability is then expressed by means of a fraction, the denominator of which gives the total number of equally likely alternatives, while the numerator gives the number of alternatives which are favourable to the event in question. The general formula may be expressed thus: $p = f/t$, where p stands for the degree of probability, t for the total number of equally likely alternatives, and f for the number of favourable alternatives.

This mode of assessing probability at once suggests quantitative expressions for certainty, as limiting cases of probability. There are two cases of certainty, namely, when we know that something is necessary, and when we know that something is impossible. We are just as certain that equilateral triangles cannot be right-angled as that they must be equiangular. But, owing to the habit of applying the terms probability and certainty to the events, instead of to our judgments about them, it is usual to employ the term "certainty" for only one of its two forms, namely, necessity, the other form being separately designated as "impossibility." One does not want to be pedantic in the use of words, and so long as it is remembered that impossibility

is also a case of certainty, in the wider sense and more correct use of the term, there is no harm in conforming to current usage. Impossibility means that none of the possible alternatives is favourable. Its probability is, therefore, o/t, which $= 0$. For example, it is impossible to throw an ace with a coin, for there is no ace on a coin, only head and tail. Similarly, it is impossible to throw head with a die, for a die has no head, only facets marked from 1 to 6. The probability is, therefore, 0/2 and 0/6 respectively, that is 0, in either case. Certainty, on the other hand, means that the event contemplated must happen, either because it is the only possibility under the given conditions, or because, although there are a number of different alternatives, yet any one of them will serve the purpose in hand, that is to say, will be favourable to the result in question. In either case all the alternatives are favourable, so that in this limiting case $p = t/t = 1$. In this way, starting from the formula $p = f/t$ we can deduce from it the limiting probability-values of impossibility and certainty as 0 and 1 respectively. So that the values of probabilities proper must be more than 0 and less than 1. In the two limiting cases it is not even necessary to stipulate that the alternatives should be equally likely.

In the case of simple events it is usually quite easy to determine the values of t and f. Thus, for example, the probability of throwing head with a properly constructed coin is 1/2. That of throwing a 6 with a well-balanced die is 1/6; that of throwing 1 or 6 is 2/6 or 1/3; that of throwing an even num-

ber is 3/6 or 1/2; and so on. In the case of complex events, however, the task is somewhat more difficult, and calls for some caution. By a complex or compound event is meant one in which two or more separate events can be distinguished. Now, the total number of possibilities in such cases does not consist of the sum of the possibilities of the separate events but of their product. Moreover, each possibility is not something simple; it is complex, and must be expressed in terms of all the component events. Both these points must be borne in mind if mistakes are to be avoided. For instance, if a die is thrown twice (or two dice are thrown simultaneously) the total number of possibilities is not $6 + 6 = 12$, but $6 \times 6 = 36$, because for each possibility with one die or throw, there are six possibilities with the other; therefore altogether there are 6×6 possibilities. Sometimes, the sum of the separate possibilities is equal to their product. In the case of two coins, for example, the sum of their possibilities is $2 + 2$, and the product is 2×2, and the two are equal. In such a case, it is especially important to remember the second of the above points, namely, the complexity of all the possibilities. Suppose, for example, we want to know the probability of obtaining head in one or other of two throws with one coin, or with two coins thrown together. We might argue plausibly that there are four possibilities, namely, head or tail on the one coin, and head or tail on the other coin—h_1, t_1, and h_2 or t_2—and that two out of the four possibilities are favourable; therefore, $p = 2/4 = 1/2$.

But that would be wrong. There are four possibilities, it is true, but they are all complex, not simple like those just given. The four correct possibilities are (1) head on both coins (or in both throws); (2) head on the first and tail on the second; (3) tail on the first and head on the second; (4) tail on both—$h_1\ h_2$, $h_1\ t_2$, $t_1\ h_2$, $t_1\ t_2$. Of these four possibilities three are favourable (the only unfavourable one being that in which tail is thrown in both), and therefore the true probability is 3/4. In the case of two dice thrown simultaneously (or two throws with the same die) the correct possibilities will be as follows, if we let the first digit in each number stand for the number appearing on the first die (or in the first throw), and the second digit for the number appearing on the second:—

11	12	13	14	15	16
21	22	23	24	25	26
31	32	33	34	35	36
41	42	43	44	45	46
51	52	53	54	55	56
61	62	63	64	65	66

Now the probability of a compound event, as compared with the probability of the separate component events, may be either greater or less, according to circumstances. If, in the case of the complex event, all that is required is any one of the component events, then the probability of the complex event is greater than that of any of its simple components. For example, we have just seen that the probability of obtaining head in either of two throws of the coin is 3/4, which is greater than that of getting it in one

throw only, which is 1/2. Similarly, the probability of getting a 6 in either of two throws of a die is greater than that of getting it in one throw only, namely, 11/36 as against 1/6 (see the foregoing table). The reason for this is fairly clear. When a second throw is permissible, then we may still get what is required, in the second throw, even if we have missed it in the first throw. Therefore the second throw increases the probability. But the permissibility of the second throw does not double the possibility of the one throw only, because the permissibility of a second throw will have no value if we obtain what is required in the first throw. The case of h_1 h_2 in the case of the coins, or that of 66 in the case of the dice, must not be counted twice.

On the other hand, the probability of the compound event may be less than that of the simple events which compose it. This happens whenever the compound event contemplated is one in which certain component events must all be there in a certain order. It is obvious, on general grounds, that the probability of both events, A and B, happening is less than that of A alone or B alone happening, because either of them might happen *without* the other, as well as *with* the other. Thus, for instance, it is less probable that head will appear in both of two throws with a coin than in one throw, because we may fail to get head in the second throw even if we get it in the first throw. Now, in such cases, the probability of the compound event is obtained by multiplying the fractions expressing the separate probabilities of the several component events. Thus

the probability of throwing head in both of two throws of a coin is

$$1/2 \times 1/2 = 1/4\,;$$

that of getting 6 in both of two throws of a die is

$$1/6 \times 1/6 = 1/36\,;$$

that of getting an even number in both throws is

$$3/6 \times 3/6 = 9/36 = 1/4\,;$$

and so on. The reason for the multiplication of the separate possibilities in this type of case may be stated as follows. The event contemplated is one in which several component events, say A and B, must all occur. Now, unless A does occur it makes no difference whether B occurs or not; it is of no interest to us then. Therefore, B will only be taken into account if A does happen. It is consequently not certain, but only probable, that B will be taken into account at all. How probable? As probable as that A will happen, say p_a. Now, if A does happen, then B is taken into account; but it is not certain that B will happen even then. There is only a probability of its happening even as a separate event, say a probability of p_b. Hence, the total probability of B happening precisely as required, namely, when A also happens, is $p_a \times p_b$. Similarly with more complicated events.

What has just been said about compound events and their being less or more probable, according to circumstances, than the separate component events, applies also to events or things whose probability cannot be calculated at all. Thus, for example, it

is more probable that the weather will be fine on Monday *or* Tuesday than on Monday; but it is less probable that it will be fine on both Monday *and* Tuesday than on Monday. Incidentally, this will explain to some extent why some hypotheses are less probable than others. The wave theory of sound, for instance, is, in the light of the foregoing considerations, more probable than is the wave theory of light, because the latter assumes two things, namely, the existence of ether and wave-transmission, whereas the former theory assumes only the wave transmission, since its medium (air) is known to exist, and is not a matter of assumption.

§ 3. *Equally likely Possibilities.*

To return to the above-mentioned conditions for the *a priori* calculation of probability. The second of those conditions calls for special consideration. First, why must the alternatives be equally likely? A comparison of two simple cases will make the point clear. Suppose we were to argue that a properly constructed coin must throw either head or not; that there are, therefore, two alternatives, of which head is one; so that its probability must be 1/2. The answer would be true, but the reasoning would not be correct. Let us now take a die instead. Suppose we were to argue that the die must either throw ace or not; that there are, therefore, two alternatives, of which ace is one; so that its probability must be likewise 1/2. Here the answer is palpably wrong. But why? Is it not true that the die must throw either ace or not? Are there not

these two alternatives? In a sense there are two alternatives in the case of the die as well as in the case of the coin. But whereas in the case of the coin the alternative "not-head" is equal to the alternative "head," in the case of the die the alternative "not-ace" really stands for five separate alternatives grouped together as one alternative—it is really five times as great as the alternative "ace," and must be weighted accordingly. Its probability is really 5/6, while that of the ace is only 1/6. In the same way, the alternative "bull's-eye" is not comparable with the alternative "not bull's-eye," for the former represents but one comparatively small space on the target, while there are innumerable places on and off the target which are "not bull's-eye." Similarly with the happy mean in right conduct, as Aristotle would say. Incidentally, this will also explain the absurdity of the view that a statement for which there is no evidence *pro* or *con* has a truth-probability of 1/2. There is no reason whatever for supposing that the truth and the falsity of such a statement are equally likely alternatives. There are innumerable ways of missing the truth, just as there are innumerable ways of missing the mark.

Next, the condition appears to be question-begging. Apparently we must know that the alternatives are equally probable before we can determine the probability of any one of them. If so, how is one ever to get a start? Poincaré, accordingly, had no difficulty in poking innocent fun at the whole calculus of probability, which he

considered to be little better than a game conducted in accordance with certain rules and conventions. But it is an exaggeration to regard the equality of the alternatives as a mere matter of convention. The equality of the alternatives may be ascertained without the aid of fallacious question-begging or arbitrary conventions. Let us suppose that we have witnessed the construction of a die made as nearly perfect as possible. What do we know in that case, and what are we justified in anticipating from a throw of the die? We know that the principle of gravitation and the laws of centre of gravity and of specific gravity are operative, and that the six sides of the die are approximately equal in shape and in weight. From all this we deduce with comparative certainty that the die when cast will not remain suspended in mid-air, or stand up on one of its edges, or remain poised on one of its corners, but must fall flat on one of it's sides. But which side? That we cannot tell, because there are six equally likely alternative sides. In what sense are the alternatives equally likely? In the sense that the conditions which are known to be operative are known to be equally favourable to any one of the six sides of the die. Other conditions, namely, the precise way in which the die will be thrown, etc., will have to become operative in order to make the die fall on one side rather than on any of the other sides. But, since these conditions are unknown, we can only be guided by the known conditions, which are equally favourable to each of the six sides. In this sense the six alternatives are equally likely. That is all that is

required by the condition under consideration, so that no convention is necessary, nor is question-begging involved. It is not always possible to ascertain the equal likelihood of the alternatives in such a direct way, and then one is tempted to let mere ignorance of inequality do service for a knowledge of equality, a procedure that is not always justifiable. The equality of alternative possibilities can sometimes be determined indirectly. Suppose, for example, we have a die of the proper construction of which we have no evidence, and which we do not want to take to pieces in order to examine its sides, we might still ascertain indirectly whether its six sides are equally likely alternatives. We might, namely, cast the die several thousand times and record the results. If, on an average, each side of the die appeared approximately once in six throws, then we should feel justified in regarding the alternatives as equally likely. Observations of the actual behaviour of the alternatives are thus made a test of the equal likelihood of those alternatives. This brings us to another method of estimating probability—the *a posteriori* (or inductive) method.

§ 4. *The Inductive Calculation of Probability.*

Many, and practically the most important, cases of probability cannot be calculated *a priori* at all; but they can be estimated by the aid of sufficiently numerous observations of the class of events contemplated. Suppose, for instance, that, in the above case of the die, the six different sides were not thrown an

approximately equal number of times—the ace, say, turning up on an average once in about eight throws, instead of once in six. This would be taken to prove that the six alternatives are not equally likely, so that the probability of any particular throw with that die could not be calculated *a priori*. But it could still be calculated by the aid of the actual observations—ace, e.g., would be said to have, a probability of 1/8. The frequency with which an event occurs, in the long run, is treated as the measure of its probability. In this way, provided we have sufficient statistical data, it is possible to estimate exactly the probability of all sorts of events which do not lend themselves at all to *a priori* treatment—births, marriages, deaths, the thousand-and-one ills that flesh is heir to, etc.

Influenced by the empirical tendencies of a scientific age, and contemptuous of the high *a priori* road favoured of theologians and philosophers, writers on Probability have been tempted to base all calculations of probability on frequencies. They would either banish *a priori* calculations altogether, or, at most, treat them as intelligent anticipations of frequencies. It is admitted, indeed, that the frequency-theory of Probability is not free from difficulties. Frequencies are apt to vary considerably with the number of cases observed. In tossing a coin, for example, the proportion of heads and tails varies remarkably according as one stops at the 50th, the 100th, the 1,000th, or 10,000th throw. One can get almost any proportion by stopping at the right moment. Hence the intro-

duction of the notion of "the long run"—*in the long run* a normal coin will throw head once in two throws. But even so, the element of arbitrariness is not entirely eliminated. As here conceived, the fundamental form of the probability-calculus is the *a priori* form, of which the *a posteriori* type is simply an inverse process. The probability that a die, which is known to be properly constructed, will throw ace is 1/6. Primarily this fraction does not refer to the average frequency with which ace has turned up or will turn up. It means that ace is one of six equally likely alternatives, one or other of which must be realized, should the die be cast. This may be ascertained accurately without casting the die at all. The said frequency may, however, be deduced from the *a priori* probability, of which it can, consequently, be made a test or an index. In the *a posteriori* calculations we simply say that the event in question occurs *as if it were* one of so-and-so many equally likely alternatives—the ace of the above-mentioned bad die, e.g., turns up as if it were one of eight equally likely alternatives. There is nothing unusual about such a use of the inverse process, or the resort to fictitious suppositions.

Our view of the calculable cases of probability (*a priori* and *a posteriori*) makes it possible, without any straining, to keep together all types of probability, quantitative and non-quantitative. In all cases of probability proper the certainty of anticipation is weakened by our knowledge of alternative possibilities. In some cases we know directly what

the other alternatives are; in others we do not, except, perhaps, in a vague sort of way. In some cases, we can, from a knowledge of the operative conditions, regard the alternatives as equally likely; in others we have to weight them in the light of experience; in yet other cases we may not be able to value them at all, or only in a very rough manner. The frequency-theory of probability, on the other hand, has really no room for the non-quantitative cases. Even in the modified form, in which frequency is taken to mean the truth-frequency of certain classes of propositions, it appears unsatisfactory. How exactly does it help one to deal with the probability of a judgment, to advise him to ascertain first the probability of the whole class of judgments to which it belongs? Strictly speaking, it is only when we know the probability of a proposition that we know to what class it belongs in respect of truth-frequency, if we can ever know this at all.

§ 5. *The Calculation of Odds, etc.*

The term "chances" is sometimes used instead of the expression "probability." At other times the expression "chances" is used as the equivalent of the term "odds.". This last expression is met with more often than the term probability in connection with problems of hazard. By "odds" we mean the ratio of favourable to unfavourable possibilities, f to non-f. It is always easy to convert probabilities into odds and *vice versa*. For $t = f$ + non-f, so that if we know p (i.e. f/t) we can

determine non-f, which $= t - f$, and so we can state the relation of f to non-f, that is, the odds. Similarly, if we know the odds, that is, the ratio f to non-f, we can determine t, which $= f +$ non-f, and so we can obtain p, which $= f/t$. For example, if we are told that the probability of an event is $11/36$ then $t = 36$, and $f = 11$; therefore non-f $= 36 - 11 = 25$, and the odds, therefore, are 11 to 25. Conversely, if we are told that the odds are 11 to 25, then $t = 11 + 25 = 36$, and $f = 11$, therefore the probability is $11/36$. Sometimes the probabilities and the odds given are those *against* the event; but this involves no difficulties whatever, if it is remembered that the probability against the event simply = non-f/t, and the odds against it are non-f to f. The probability against an event must be distinguished from its improbability. The expression "improbability" simply means a low degree of probability. Popularly, the term "probable" often means "very probable," or "more probable than not," and the term "probability" means "a high degree of probability."

It is sometimes easier, or quicker, to calculate the probability against an event than the probability in its favour. In such cases, if we want to ascertain p it is best to determine non-p first and to subtract it from 1. For $1 - p =$ non-p, because $p = f/t$, and non-p = non-f/t; therefore, $p +$ non-$p = f/t +$ non-$f/t = t/t = 1$ Consequently, $p = 1 -$ non-p, and non-$p = 1 - p$. For example, the probability of tossing head in either of two throws of a coin is, as was shown above, 3/4. Now, there was some

risk of miscalculation in that case, and the risk is much greater in more complex cases. It would have been simpler and safer to argue that the probability *against* obtaining head in either throw is the same as the probability of getting tail in both, that is $1/2 \times 1/2 = 1/4$. Thus, non-$p = 1/4$, and p therefore $= 1 - 1/4 = 3/4$.

§ 6. *The Law of Succession and Induction by Simple Enumeration.*

There is one special type of *a posteriori* calculation of probability that calls for separate consideration, as it bears on the question of induction by simple enumeration. If an event has been observed to occur a certain number of times under certain circumstances which do not appear to be causally connected with it, then, of course, we cannot feel confident that it will always occur under those circumstances; but the more often it has been observed to occur, the more probable will its recurrence under those circumstances appear to be. According to Laplace, each previous occurrence may be regarded as a reason for expecting its recurrence, and each failure is a reason against expecting its recurrence. Now, the question of the next recurrence of the event considered by itself involves two possibilities, namely, its occurrence and its non-occurrence. Of these two possibilities, one is favourable. Accordingly, if an event has occurred m times and has never been known to fail under certain circumstances, then the probability of its

next recurrence, under those circumstances, is $\frac{m+1}{m+2}$, or the odds in its favour are $m+1$ to 1, so that the larger the number of occurrences observed, when no exceptions are known, the more nearly does the probability of the next occurrence approximate to 1, that is, certainty. Thus, for example, if the sun has been observed to rise and set within periods of twenty-four hours on a trillion occasions, and there has been no exception, then the probability of its rising and setting in the next twenty-four hours is $\frac{\text{a trillion} + 1}{\text{a trillion} + 2}$! which is practically 1, or certainty. But the probability of more than one such recurrence is certainly less. The probability of r recurrences will be $\frac{m+1}{m+r+1}$. Induction by simple enumeration may thus attain to a high degree of probability if the expectations based upon it are confined to a comparatively small number of recurrences. But the probability of a real generalisation based upon simple enumeration is never very high. For in a real generalization the number of recurrences contemplated (r) is practically infinite, and consequently the value of $\frac{m+1}{m+r+1}$ cannot be high.

If, under the same circumstances, the event in question has been observed m times and has failed x times, then the probability of r recurrences will be $\frac{m+1}{m+x+r+1}$. This formula is known as the

Law of Succession, and includes, of course, the preceding formulæ, which are obtained from it if x or $r = 0$, according to the nature of the case. In this fuller form we see the basis of the probability of the recurrence of partial correlations or associations, as ascertained by the statistical method of exact enumeration. What has just been said about the probability of inductions by simple enumeration applies here likewise.

§ 7. *The Use of Calculations of Probability.*

Lastly, of what practical value is the whole calculus of probability? Some people have very exaggerated notions of its practical value. This is partly due to our respect for figures, in consequence of the important place which mathematics holds in modern science. One tacitly assumes that exact figures express precise knowledge, and one has no suspicion of the ill-defined nebulosities that may masquerade in precise ratios and fractions. Partly, however, the exaggeration is also due to the frequency-theory of probability. This theory rather encourages the confusion of probabilities with frequencies. There is, of course, a connection between them. We have seen that probability can, in many cases, only be calculated on the basis of observed frequencies. Nevertheless, probability and frequency are not identical. Now frequencies, when treated with the necessary precautions, may be of the greatest practical value—as is evident from their use in insurance business and in kindred enterprises. But the calculus of probability is another

matter. Probability is concerned even with single events and small groups or series of events, while frequencies always refer to large classes, or long series of events, to what happens "in the long run." The practical difference is obvious, even in roulette. The bank, doing business with large numbers of players, can rely on frequencies which secure it certain advantages. The individual player, limited to a much smaller number of ventures, simply gambles, and usually pays dearly for his estimates of probability, even when these are based on rational calculations, and are not merely the result of inspiration or superstition. The calculus of probability is, of course, secure of its reputation, just like the ambiguous oracles of antiquity. Whatever happens, the calculus is right. Whether you win or lose, whether you have a long run of good luck, or of ill luck, the calculus is equally right. This may console those who cherish their delusions more than their treasure; but sensible people will not put their trust in such ambiguous oracles. There are, it is true, ingenious gambling systems, based on frequencies rather than on probabilities. But even these systems have their day and cease to be. The best of them is but a snare, cheating the fowler of a bird in the hand for two in the bush. Its validity always depends on "the long run," which easily outruns the resources of any ordinary individual. For similar reasons, even in legitimate insurance business, the company, relying on frequencies, is on a much better footing than the individual client. But the practical exigencies of life often make it

advisable for the individual to take high risks for small amounts rather than incur low risks for large amounts—not to mention the benefits which accrue to the community as a whole from insurance organizations. No precision of figures can eliminate the essential uncertainty of the probable. And, in the last resort, the best method of estimating the probability of anything is by a close examination of the actual conditions. Even insurance companies do not rely entirely on frequencies, but have each case examined by an expert (doctor, or engineer, etc.), according to the nature of the risk. This is done primarily in order to determine accurately to what precise class the risk belongs, since each class has its own frequency. But that is not the whole explanation. For, if everything were taken into account indiscriminately, each case would be *sui generis*. There can be no doubt, however, of the great practical, as well as theoretical, value of a knowledge of frequencies and correlations. In comparison with it, the value of the calculus of probability is almost negligible.

§ 8. *Probability and Frequency.*

The relation between those kinds of events the probability of which can be calculated deductively (or *a priori*) and those events the probability of which can be calculated only inductively (or *a posteriori*) may be indicated in yet another way. In the deductive type of cases what is really calculated in the first instance is, as has already been explained, the probabilities or possibilities, not

actual frequencies; but from these possibilities or probabilities we infer the frequencies; and the same fraction is used to express both the probability and the frequency. Yet the two are not identical, as may be seen from the different meaning of the same fraction according as it is intended to express the probability of an event, or its frequency. For instance, the fraction 1/6 when it expresses the *probability* of face 6 turning up when a properly constructed die is thrown, simply means that "the turning up of face 6 is one of six equally likely possibilities"; but when this same fraction is used to express the *frequency* of the event in question, then it means "*in the long run* face 6 will turn up on an average in one out of six throws." The phrase "in the long run" is irrelevant in the case of probability strictly so called; but it is indispensable in the case of statements of frequency.

In the case of the deductive calculations, then, the frequencies are really deduced from the probabilities. On the other hand, in the case of the inductive calculations what we start from is the statement of frequencies actually observed and recorded. And from these frequencies we infer the probabilities by an inverse process, as has already been explained. And in such cases the fraction representing the frequency of the event "in the long run" is translated into an estimate of the probability of, it may be, a single event, by a kind of legal fiction, "as if" the event in question had *f* possibilities in its favour out of *t* equally likely possibilities.

CHAPTER XXIII

ORDER IN NATURE AND LAWS OF NATURE

§ 1. *Order in Nature.*

In the preceding chapters reference has been made repeatedly to the order, laws or regularities which science seeks to discover among the phenomena of nature. A few explanations are now called for. When we speak of order or orderliness in nature, or in the world, what we mean, in the first instance, is the opposite of chaos or mere chance. In daily life we have frequent occasion to describe certain occurrences or concurrences as merely chance incidents or coincidences, while other occurrences are not treated as mere matters of chance. When dealing with human actions, the opposite of chance or accident consists of what is usually called design or purpose. Sometimes, for example, we meet friends by chance, at other times designedly, or of set purpose. But, when investigating the vast majority of natural phenomena, we are not concerned with problems of design or purpose. Here the opposite of chance is usually referred to as necessity. which must not, however, be taken to mean the same as compulsion, but simply conformity to a natural law or regularity of some kind. There are two questions then that have to be considered: (*a*) What is the attitude of science towards order in nature? and (*b*) what is meant by natural laws and regularities?

Science, in seeking to discover order in nature,

would appear to assume that there is such a thing. To some extent this is true. If there were no order in nature there would be nothing for science to do. Certainly science does not propose to invent order in nature, or to introduce order into nature, but only to trace it, to discover it, if possible. At the same time, the search for anything does not really or necessarily presuppose the definite assumption or conviction that what is sought is actually there. One may look for what one merely hopes to find, or for what one considers to be there more or less probably. Again, to consider it more or less probable that there is order in nature, is not the same thing as to assume that nature is orderly through and through. After all, the world is vast, and the field of scientific investigation is, in comparison, very limited. The man of science can always select for his investigation a class of facts in which the discovery of order seems promising.

On the whole, experience has shown that there is some order in nature, even if nature be not orderly through and through. If there were no order at all in the world, if the actual distribution of things and attributes and the occurrence of events were entirely a matter of chance, then there would be nothing to exclude any conceivable, or even inconceivable, combinations of attributes or sequences of events. For order expresses itself through laws or regularities. The whole order of things in any system, whether in nature as a whole, or in any part of it, really consists of a system of inter-connected laws, which constitute as it were the threads of its orderly pattern. Now

a law can be expressed in a universal proposition of the form "If S is M, it is P," or, more briefly, if not quite so accurately, in the form, "S is P." And each of these forms of expressing a law excludes certain conceivable combinations, namely, things which are S and M but non-P, in the former case, or things which are S and non-P, in the latter case ($SM\bar{P} = 0$, or $S\bar{P} = 0$, where $\bar{P}$ stands for non-P).

The complete absence of natural laws, or what comes to the same thing, the complete absence of order in nature would, therefore, show itself in an absence of exclusions of any conceivable combination of attributes or events. What, however, experience plainly shows is that many such conceivable combinations and sequences are not met with. On the other hand, many laws have been discovered which, if true, would definitely exclude various conceivable combinations, the absence of which they may be said to account for. And a considerable amount of experience has so far confirmed these laws. Moreover, the larger the number of laws which science succeeds in discovering, the greater does our confidence become in the extensiveness of the domain of the reign of law, or rather of the pervasiveness of law. True, even the aggregate of human experience is comparatively limited, and our discoveries are always subject to subsequent correction in the light of further experience. There is no finality in human knowledge, not even in scientific knowledge. But sufficient for the day is the evil thereof. So far the greatest discoveries of the past have not

had to be entirely repudiated, only corrected or reformulated.

At the same time, there is *primâ facie* evidence enough of disorder and chance in the universe. Not only are there vast regions of fact in which, as yet, no order has been discovered, but even among the orderly facts or events there are apparently elements of disorder. The individual members of any class of phenomena, for example, usually show considerable deviation from the type, as has already been pointed out in Chapter XVII, and the most careful measurements of changes subject to natural laws almost always show deviations from one another and from the adopted, or so-called true, value. These deviations (or "errors," as they are often called) cannot be entirely explained away on the ground of human incompetence. They seem to point rather to an element of lawlessness (originality or spontaneity, if you like) in the facts or events themselves.

We may conclude, then, that there are laws in nature, although these laws are not always rigid, but somewhat elastic. There are, however, also parts of nature in which, as yet, no laws have been discovered, and where, so far as one can tell, laws may possibly never be discovered. If we are to use the familiar expression "Uniformity of Nature" to describe the general character of nature, as just discussed, it must not be taken to mean more than that there are laws or uniform connections in nature, or in many phenomena of nature; and it must not be taken to mean that such laws are absolutely rigid,

or that all the phenomena of nature are subject to laws. Least of all may it be interpreted in the sense that the *course* of nature is uniform, or that natural events proceed like recurring decimals. Even if nature were orderly through and through, there would be no need for a perpetual cycle in all natural events. Uniformity of nature is something quite different from the uniform course of nature.

It is possible that order in nature is itself the result of evolution, that in the remote past there may have been even less order than there is now, perhaps no order at all, and that, at some remote future time, nature will exhibit more order than it does now. If so, it would be all the more intelligible why man always looks for order in nature; why he is for ever seeking to discover laws and regularities among natural phenomena, and more so now than he did in the past. His whole attitude, namely, may be the outcome of a growing adaptation to his environment. But this is only a highly speculative suggestion.

§ 2. *Natural Law.*

What is a natural law? Most people are far more familiar with the legal or moral use of the word "law" than with its scientific use, hence a certain amount of confusion. In the legal or moral sense, the expression "law" means a command backed by sanctions and penalties. Usually also, in these cases, the laws are considered to be imposed upon us from outside by some authority, such as the State, Society, etc. It is quite different from what

is called a "natural law." In its scientific sense the word "law" means nothing more than a regularity or uniformity in the character or relation of certain classes of facts or events. It denotes just some intrinsic character, or mode of behaviour, in certain classes of phenomena—nothing imposed on them from outside, but just an orderliness of their own nature.

Another confusion to be guarded against is that between what may be called the law itself, that is to say, the actual objective regularity of the phenomena in question, and, on the other hand, the verbal or symbolic formulation of the law, that is to say, the formula. Unfortunately, the term "law" is frequently used ambiguously, sometimes for the uniformity itself, and sometimes for the formula. What is true of the one may not be at all true of the other. The formulation of a law is the work of men of science. It may require correction from time to time in order to make it express the law or uniformity itself more accurately in the light of increased knowledge. The formula may even have to be rejected altogether, if subsequent research gives ground to suppose that there is no objective law or regularity corresponding to it. But the law or uniformity or regularity itself is usually assumed to be independent of its discovery and formulation. The man of science does not endeavour to invent it, but only to discover it. The formula, moreover, may only be a close or rough approximation to the law, and in any case it is not the law itself. Natural laws, in short, are not man-made, but only man-

discovered and man-formulated. Whatever one's view may be of the ultimate constitution of the universe, or of nature, even if one were to conceive it on idealistic lines, it would be a distortion of the ordinary scientific view of natural laws not to regard them as objective, that is to say, as independent of the discoverer or investigator. We may admit that the scientific formulæ of natural laws are often only approximations, sometimes even deliberate simplifications; but this admission, so far from weakening, actually strengthens the case for conceiving natural law as something strictly objective, a part of nature itself, which the formula does not always express completely.

There are various kinds of natural law. Some of them appear to be more rigid, others less so. Some are almost invariable, others are what may be called regularities rather than more or less rigid uniformities. It is always possible, however, that the inexactness or elasticity of certain natural laws, as conceived by us, may be due to human shortcomings, rather than to the laws themselves, which may simply not be adequately expressed as yet. More important is the distinction between laws of co-existence and laws of sequence. There is a law of co-existence, whenever a number of attributes or states are regularly together. In the case of natural classes, for instance, certain characteristics are usually found together. Similarly, with various physical states, or the properties of various kinds of geometrical figures. A law of sequence consists of the regular successions of certain states or events,

as, for example, between changes of temperature and changes of volume, between thunder and lightning, between the seasons of the year, and so on. Again, some laws are said not to be causal, or are not known to be causal, but are either rational regularities, or merely regularities which may, nevertheless, so far as we can tell, be reducible eventually to as yet unknown causal laws. But the distinction between causal and other regularities, although interesting and really justifiable, or even inevitable, is of no special scientific importance. The really important thing is to discover regularities, of whatever sort. Regularities make the world orderly, and a knowledge of them makes the world, or at least a part of it, intelligible and manageable. For the knowledge of a general law spares us the need of worrying over each particular case. That is an important thing. The question about what may be called the machinery of these regularities is interesting, of course, and we shall, therefore, consider it briefly, but it is only of secondary importance in science.

Another distinction is that between ultimate or primary, and secondary [1] or derivative laws. An ultimate or primary law is one which cannot be deduced from any other law or laws. A derivative or secondary law is one which can be so deduced, or which is not sufficiently comprehensive to be regarded as primary. For example, Kepler's three laws, already referred to above, are derivative,

[1] The terms "primary" and "secondary" are here used more or less in the same sense as in the distinction between primary and secondary qualities.

they can be deduced from the law of gravitation; but the law of gravitation itself is, so far as we can say at present, an ultimate law. The term "ultimate" in this connection must be understood in a relative sense, just like the term "element" in Chemistry. In both cases alike the meaning of the term must be qualified by the reservation "in the light of present knowledge." The expression "law of nature" is sometimes confined to the so-called ultimate laws, but in a wider sense, of course, all natural laws are laws of nature.

§ 3. *Condition and Cause.*

The facts of observation are extremely complex, and in our attempt to trace order in them we have, as it were, to unravel the tangle and follow up the separate threads. That is to say, in every object or event that we observe scientifically, we regard certain of the constituent factors as connected with one another, while others are not considered to be connected with these same factors, though they may be connected with different ones. For example, in the case of falling bodies, we regard the time of the fall as connected with its velocity, but we do not regard the velocity as connected with the chemical constitution of the body, though its chemical constitution will be conceived to be connected with other things, such as its reaction to various agents, and so on. Similarly, in the case of triangles, we regard the fact that the sum of the three angles is equal to two right angles as connected with the bare trilateralness of the figure, but not connected

with the relative lengths of the three sides, though the relative lengths of the three sides will be considered to be connected with something else, namely, the relative sizes of their opposite angles. Now the facts or factors may be said to be connected with one another if one of them is impossible without the other; or, what amounts to the same thing, if one of them is a condition of the other. By a condition of anything, then, is meant whatever is indispensable to that thing or event, etc., or that in the absence of which the thing will not be, or the event will not happen, or at least will be different. Thus the rotation of the earth is a condition of the sequence of day and night. A certain kind of structure of the stomach of an animal is a condition of its ability to chew the cud, or the equilateralness of a triangle is a condition of its equiangularity. The result, or whatever it is that the condition renders possible, is called the consequent, and every connection may accordingly be described as a connection between a condition and a consequent. Sometimes, as the last example may illustrate, condition and consequent are reciprocal, that is, either renders the other possible; but this is not always or frequently so. Most results are complex, and need the fulfilment of a number of conditions, each of which is indispensable, but is not adequate by itself to complete the result. Thus, for example, the form of a certain curve, the trajectory of a bullet, or the orbit of a planet is the result of a number of different co-operating conditions.

It should be noted that sometimes the condition

and the consequent are simultaneous, and their connection constitutes what has already been described as a law or uniformity of co-existence. In such cases, of course, the term "consequent" must not be interpreted in any temporal sense, but only in a logical or quasi-logical sense, inasmuch as the knowledge of the uniform co-existence of certain attributes enables us to infer from the presence of one of them also the presence of the other. With uniformities of sequence it is different. In their case the consequent really does follow the antecedent, though the connection between them is less frequently reciprocal than in the case of uniformities of co-existence.

Moreover, in the case of the consequents which follow their conditions, we have to take into account negative conditions as well as positive conditions. By a negative condition of any result is meant the absence of whatever may thwart the appearance of that result; while other conditions, the so-called positive conditions, may be said to contribute positively to the result, or to constitute it more or less. Thus, for example, the velocity of falling bodies is conditioned positively by the time of their fall, and not by their mass. Two bodies of different mass or density, say snowflakes and hailstones, should, therefore, fall through the same distance in equal times; but that will not be the case if they fall through a resisting medium, such as air, etc. Now, the absence of a resisting medium is, therefore, the negative condition of the result contemplated, namely, the fall of different bodies, having different

masses or different densities, through equal distances in equal times. Similarly, in the economic sphere, supply and demand and the price of commodities are inter-connected in certain ways, but the actual results may be thwarted or masked by Government control, etc. The absence of such control, etc., is, therefore, a negative condition of the results considered.

Certain sequences of events are commonly considered to be causally connected either directly or indirectly. For instance, an increase of pressure and a fall in temperature would be said to be a direct cause of diminution in the volume, or an increase in the density, of a gas. On the other hand, the seasons, though they follow each other with a certain uniformity, would not be said to cause one another. Rather their sequence would be described as the indirect result of the movement of the earth in relation to the sun, as determined by gravitation and the relative masses and distances of the planets. In this case, a certain uniform sequence, and in other cases, such as those of various kinds of plants, animals and chemical elements, certain uniform co-existences, would be regarded as indirect results of certain causes. By the cause of an event or result is meant the minimum totality of conditions each of which is indispensable and all of which are together just sufficient to bring about that result. It should be pointed out, however, that commonly the expression "cause" is used in the sense of "condition," or part-cause.

What distinguishes causal connections from other

uniformities and regularities is this. The causal conditions are believed to supply the matter and the energy of their consequent or effect; in other words, the event which is the effect of a given cause consists of the matter and the energy of the several conditions composing the cause. The cause has simply been transformed into its effect, or each condition in the cause has been transformed into some part of the effect. The difference between the character of the effect and that of the several conditions, which together cause it, is the result of the action of the several conditions on one another. Now the case is different here from other kinds of uniformities, whether of co-existence or of sequence. In the case of co-existing attributes, the attributes all remain together, there is no question of the conversion of one of them into another. Similarly, with non-causal uniformities of sequence. The seasons, for example, are not conceived to produce one another, or to be transformed one into another. On the other hand, each season considered in relation to its cause is just the result of the light and warmth, etc., which stream upon the earth, when the earth and sun are in certain positions relatively to one another. Thus regarded, it seems legitimate enough to distinguish causal uniformities or laws from others. In the case of physical phenomena they constitute a relation of greater unity and continuity than do those other uniformities which are not causal, either directly or indirectly. Our knowledge of causal laws in any sphere of inquiry, therefore, facilitates a higher degree of coherence and system

in our knowledge of natural phenomena, so as to approximate to some extent to the logical coherence of the mathematical uniformities.

There are some thinkers, however, who object to the use of the term "cause" in science. The idea of causation is derived from the experience of human exertion or effort, and the term "cause" may suggest to some people that the so-called causes, or causal conditions, exert a similar effort in producing their effects. That would be anthropomorphism pure and simple, if not fetishism, or what not. Certainly, such confusions should be avoided as far as possible, but the fear need not be carried to excess. Language is the creation of man, and the tendency to anthropomorphism is its original sin. If science is to avoid every suggestion of anthropomorphism, it must be dumb. Does not the word "law" sin in the same way as cause? And energy? Even the word "routine," a favourite substitute for causation, may be just as misleading. The important thing to guard against is excessive anthropomorphism, whatever terms we employ. After all, it may be possible to carry this fear of anthropomorphism to excess, for man is also a part of nature. Another reason for the objection to the recognition of the causal relation in science is to be found in our inability to see exactly how the so-called causes produce their effects. But even if the accuracy of this reason is admitted, it does not seem to justify the rejection of the recognition of causal relationship. Does any sensible person deny that he sees the heavenly bodies merely because he does

not understand how he does it? However, as already remarked, the important thing is to discover uniformities and regularities in nature, the finer distinctions between them are of secondary importance. In some ways, there is a greater simplicity and continuity in the conception of all regularities and uniformities as constituting a continuous series of correlations from the most imperfect to the most complete. The reduction of the conception of causal connection to that of mere law, or routine, or uniformity may itself be regarded as an expression of the general tendency of modern science towards greater and greater simplifications. On the other hand, the recognition of the causal relationship, as formulated above, seems to be a necessary aid to the conception of the continuity of natural events, and a valuable adjunct to the Principle of the Conservation of Matter and (or) Energy. In the absence of some such conception of causal continuity, each succeeding state of nature would appear to follow the preceding state by a kind of miracle—as, indeed, some of the Schoolmen and others thought it did.

§ 4. *The Principle of Fair Samples.*

All scientific methods start from observed facts, and usually end in generalizations of some kind concerning whole classes of facts, or events, or, at the very least, concerning large groups of them. The number of phenomena actually observed is usually a small one, in comparison with the whole class or group, of which the results of the application

of the methods are held to be true more or less. The question, accordingly, arises, what right have we to apply the results of our observation of a very limited number of facts to others, which have not been observed at all? The answer is that, strictly speaking, we probably have no *a priori* right to do so. Science in this, as in other respects, follows the lead of the practical man. In business it is often impossible or impracticable to examine each item in a large cargo of goods, such as grain, fruit, etc. Buyers are consequently content to estimate the character of the whole cargo by the aid of a sample. They are sometimes taken in; but, on the whole, experience shows that, provided a sample is selected with some care, it fairly represents the whole, although it is but a small fraction of the whole. The precautions to be taken in selecting a sample by which the whole will be judged, is mainly this: to avoid the likelihood of one-sidedness or bias. For example, a sample selected from one or two bags of grain, or cases of fruit, is not so likely to give an approximately correct idea of a large cargo consisting of many thousands of such bags or cases as when the sample is selected from a considerable number of bags or cases from many different parts of the cargo. Such a selection is commonly described as a *random* selection; but in many cases, perhaps in most cases, the so-called "random" selection calls for a good deal of forethought and insight as to the best way of avoiding bias of any kind. However, samples can be so selected; and, whenever that happens, experience

has shown that the character of an adequate or "fair" sample represents the character of the much larger whole with approximate accuracy. Of course, the result of sampling is not absolutely reliable, it can only be regarded as more or less probable, and the degree of its probability will vary with the number and the range of objects constituting the sample. The actual *number* is perhaps less important than the range of selection, that is, the *variety* of objects, wherever there is a *primâ facie* reason to suspect considerable variety in the whole which is to be judged by the sample. Much will depend on the extent of our previous knowledge of the kind of phenomena concerned. In some cases a single instance observed under suitable conditions (such as those characterizing the Method of Difference, for instance) may constitute a fair sample, in other cases a very large sample may not inspire much confidence in its fairness.

As remarked previously, the very attempt to discover a regularity of some kind presumes at least the hope or probable belief that such a regularity is there in the phenomena in which we seek it. To that extent, as already explained, we rely more or less on the principle of the uniformity of nature, in the sense previously indicated. But that principle affords no guidance whatever as to what attributes or variables might or might not be regularly correlated. In actual practice, in science and in practical life, we rely on what may be called the Principle of Fair Samples, that is to say, the belief that, with reasonable care, it is possible to

judge the character of a large group, or of a whole class of phenomena, by the aid of a sample, or a selection, from it. This principle is sometimes called the Law of Statistical Regularity.

CHAPTER XXIV

SCIENTIFIC EXPLANATION

§ 1. *Explanation and Description.*

Modern men of science appear to be fairly unanimous in maintaining that the object of science is *not* to explain the phenomena of nature, but to describe them. This view is often felt to be rather disappointing, and many thinkers have disputed its accuracy. Sometimes, indeed, men of science not only admit that science does explain, but even maintain that the scientific explanation is the only true explanation of facts. They are usually careful, however, to add at once that in science the word "explanation" means something different from what it means elsewhere. The important thing, obviously, is to get at the actual facts of the case.

The difficulty arises from the fact that things are explained differently on different occasions. The only way to indicate the general function of explanation in such a way as to include all, or nearly all, methods of explanation is to say that anything is explained when it is shown in its relation to some other thing or things, so that it does not appear, so to say, to hang in the air, detached and isolated. (The word "thing," like the word "phenomenon," is here used in the very widest sense so as to include also attributes, events, etc.) The kind of explanations with which man is most familiar are explanations of the conduct of his fellows, for they are the most important for his well-being. Now, the usual

way of accounting for human actions is by relating them with, or referring them to, some motive or purpose. When certain human actions are seen to be the means, or the steps, to the realization of some purpose, then we understand them. Our guide to such interpretation is, of course, our own felt experiences on analogous occasions. Such explanations are the explanations with which we are most familiar, and which, perhaps for that very reason, we find most satisfactory. No wonder that man always sought, and sometimes still seeks, such explanations, even when the things to be explained are not human actions. Hence the animism, fetishism, and anthropomorphism in the early history of human thought, and the cheap finalism of even some eighteenth-century thinkers, who seemed so familiar with the intentions of the Almighty! Now, modern science, except in the study of specifically human, and certain other biological problems, does not attempt explanations of this kind at all; that is to say, explanations referring to purposes, or, as they are still sometimes called, *final* causes. In the light of this, it should be clear what is meant when it is maintained that modern science is not concerned with the question *Why?* but only with the question *How?* Science, it is maintained, only seeks to discover what attributes things have, and *how* things happen, not *why*, that is, for what purpose, things are as they are, or events happen as they happen. And if the term "description" be used for any account of what things are like, and how events happen, then science may be said to be

concerned with description. That it is concerned with description is beyond dispute. The only question is, whether it is not also concerned with explanation. If the term "explanation" were to be confined to the type of explanation by reference to purpose, or final causes, then science (allowing for the exceptions just mentioned) might be said not to be explanatory, only descriptive. But then explanation by reference to final causes is not the only type of explanation. There are other types of explanation; and it is, indeed, the chief business of science to discover them. Things, attributes and processes, or events, may be explained by reference to their classes, or their conditions, or laws and regularities; and laws and regularities may be similarly explained by reference to other more comprehensive laws, from which they can be derived. Now, we have already seen that the methods of science are directed to the discovery of classes, regularities and laws. To sum up the whole work of science as description is a very inadequate way of indicating its aims and its achievements. At the very least, it is misleading, for the term "description" must, in that case, include all that is usually called "explanation," and the two terms are no longer antithetic.

The idea that science should confine itself to description can be accounted for to some extent in yet another way. This idea, namely, may be said to mark the climax of a certain revolt, clearly voiced by Francis Bacon and many others in the sixteenth and seventeenth centuries, and to some extent

already anticipated by Roger Bacon in the thirteenth century, against the extensive preponderance of speculative theory over actual observation. In the struggle of those early days it was urged, and rightly urged, that science must rest on the observation of facts, and not on the theories of authorities. The only authority for science must be the observed facts, and the rational interpretation of the observed facts. In the positivism of the eighteenth century this tendency reached its extreme form in the demand that science should shun philosophical as well as theological authorities, should shun, in fact, theoretical speculation altogether, and confine itself to the description of actual observations. The watchword of positivism has met with almost universal favour among men of science. The spirit which prompted this whole tendency was healthy. It is right that science should keep as close as possible to observed facts, and not indulge in unnecessary speculation. But this tendency may also be carried too far. What is commonly called observation includes, over and above actual sense-elements, not only such supplements of memory and imagery as make wool look soft, or ice look hard and cold, but also distinct elements of interpretation. This is not usually noticed, because the interpretation is so rapid and spontaneous that the sense-elements and the interpretations coalesce into one experience, the whole of which appears to be given immediately. That there is an element of interpretation even in observation becomes clear in the case of conflicting descriptions of the same objects or events, as frequently

happens in the Law Courts. For example, in a certain trial, one witness maintained that he had noticed on the seashore in the moonlight a woman with a child, while another witness of the same scene was equally certain that it was a man with a dog. For the most part the rapid interpretations we make in ordinary everyday life are correct. We do, indeed, make mistakes sometimes, and grow more cautious in time; but we could not check the process entirely without paralysing our intellectual and practical life. Now, what is true of ordinary experience is true more or less also of science. Description, if it is to be sharply distinguished from explanation, should be confined to what is actually observed. In reality, however, it is as difficult to separate entirely description from explanation as it is to separate entirely observation from interpretation. For scientific explanation is a form of interpretation, or rather consists of several forms of interpretation. The contention that science is not concerned with explanation at all can only be misleading, and may be partly responsible for the occasional tendency to smuggle explanations into descriptions, or to pass off theories for facts. The distinction between fact and theory, as commonly made, is, indeed, only another form of the distinction between observation and interpretation, or between description and explanation. Strictly speaking, if the distinction is to be carried through consistently, the term "fact" should mean what can actually be observed, as distinguished from the theory which links up or explains the observed facts. But only

too frequently the word "fact" is applied to anything, even a highly abstract theory, if it is believed with as much conviction as anything that is perceived ("fact" in the stricter sense). The only consistent course is to recognize that science is concerned with interpretation as well as with observation, with theories as well as with facts, with explanations as well as with descriptions; and to admit, besides, that it is not always easy or necessary to distinguish sharply between the terms in each antithetic pair.

§ 2. *Types of Explanation.*

The aim of science is to discover order in the world, and all the scientific methods are methods of tracing order among various natural phenomena. In so far as science succeeds in its enterprise, the world, or at least some part or aspect of it, appears to us more orderly or intelligible, or more explained. To explain anything is to see, or to indicate, its place in some order of things or events. Hence various methods of scientific explanation, corresponding more or less to the principal scientific methods of discovering order in nature. The chief types of explanation may be enumerated and illustrated as follows :—

(*a*) *Reference to Class.* An object is sometimes explained when it is recognized, or shown, to be a member of a known class. Thus, for instance, to one who is in doubt about the character of a certain plant, it will be explained if (whether by the aid of an analytical key, or through the help of an expert)

he finds out that it belongs to such or such a species or variety. Similarly, a class of objects may be explained when it is recognized as a sub-class of a wider class. Events also are sometimes explained in this way, as, for example, when lightning is classed with electrical phenomena. Of course, to anyone who knows nothing about the kind of objects, or class of events, to which reference is made, this is no real explanation.

(*b*) *Reference to Evolutionary Series.* A type of object is sometimes explained by reference to its place as a link in an evolutionary series. (It makes no difference whether a type is represented by the fossil remains of a single member, say, the South African skull of a primitive ape, discovered by Prof. Dart, or by a multitude of surviving members.) In such a case something which is at once similar and yet dissimilar, when compared with other types, and therefore puzzling, is assigned its place in a certain order of continuous development, and is thus explained. Similarly, a whole evolutionary series may be explained by ascertaining or indicating its place in a more comprehensive evolutionary series.

(*c*) *Reference to Mediating Factors.* When the problems concern apparently remote or diverse facts, or events, which, nevertheless, appear to be connected, then an explanation may take the form of discovering, or indicating, intermediate factors or events, which bring the correlated, but remote, facts or events into closer connection. Thus, for example, the perception of sound is explained by the mediation of air-waves between the source of

sound and the hearer. Similarly, the perception of luminous objects is explained by the mediation of ether-waves between the luminous object and the seer. And the correlation between the presence of cats and the abundance of clover is explained by reference to such intermediate events as the cats' destruction of the mice that would destroy the bees, which fertilize the clover.

(*d*) *Reference to Laws*. The commonest type of explanation consists in referring what needs explanation to some relevant law or laws. The laws may be partial or complete correlations, and, if complete correlations, they may be merely empirical uniformities or causal or logical connections. Thus the frequent occurrence of suicides in a certain city, or country, may be explained, after a fashion, by reference to the approximately constant rate of suicides there. The bent appearance of a stick partly immersed in water may be explained by reference to Snell's Law of Refraction. The successive positions of a planet may be explained by reference to Kepler's 1st or 2nd Law, or both. The movements of a planet, or Kepler's Three Laws, might be explained by reference to the Law of Gravitation. Lastly, the equiangularity of a triangle may be explained by reference to its equilateralness, or *vice versa*. The most satisfactory scientific explanations are those based on causal or logical uniformities. Other laws, indeed, are felt to need explanation themselves, and the attempt is usually made to explain them by reference to causal laws, or theories.

(*e*) *Reference to Purpose.* In the study of certain biological phenomena, and, above all, in the study of human experiences and activities, individual and social, it is scarcely possible to dispense with the conception of purpose, if we are to have really adequate explanations. Even the most violent opponent of teleological explanation, even the most thoroughgoing determinist, would hardly be flattered if his writing and other activities were described as guided by no aim, and devoid of all purpose! Still, even these special problems must be explained in other ways, as well as teleologically; and in the case of the purely physical sciences teleological explanation has no place.

§ 3. *Theory and Law.*

Laws, if they can be explained at all, are explained by reference to wider laws, as has already been pointed out. But the converse is not always true. The reference of a law to a wider law does not always yield an explanation. In order to make this point clear it is necessary to distinguish first between real inductions and mere summaries. If, after examining a sufficient number of various animals having cloven hoofs, and noticing that they are ruminants, we conclude that all animals with cloven hoofs are ruminants, then we have an induction or generalization. It may, or may not, be justified, still it is a real inference from observation to a law or uniformity. But if we confine ourselves to the statement that "all the anmals with cloven hoofs which we have examined were ruminants," then we have

merely a summary account of the actual observations, which involves no such inference as the previous statement. It formulates no law. Similarly with statements made about a limited group or class after examining each member separately. I mean such statements as "No month has 32 days," or "All the Apostles were Jews." These are merely summary statements. Such summaries are very useful as aids to memory, and for purposes of easy reference; but they are not inductions, although they have been described as *perfect* inductions, that is to say, inductions based on perfect (= complete) enumeration. Now, laws, too, may be summarized in such a manner, and when they are so summarized, the result may be called a summary law, and will, of course, be more comprehensive than any one of the laws which it summarizes. But it is not a new induction, for it includes nothing that is not already included in the laws which it summarizes. For example, if, after experiment with some samples of a particular gas, it is inferred that in all cases of that kind of gas the pressure and the volume vary inversely, if the temperature remains constant, then there is an induction. But if, after making similar inductions about each known kind of gas in turn, it is asserted that "the pressure and the volume of any gas vary inversely" (Boyle's law), then the statement is a summary of the separate inductions; it is not an additional induction. Similarly, if the Law of Refraction of Light (or the Index of Refraction) for each kind of transparent medium were ascertained first, and then all

the separate inductions were summed up in Snell's Law of Refraction. It might be urged that such summary laws (like Boyle's Law or Snell's Law) are really genuine inductions, because they also apply to certain kinds of substances which may be discovered in future (say, new gases, or new refractive media). But it is not very likely that such laws would really be assumed to hold good of newly discovered substances without experimental verification. This means that the relevant summary law would be treated as an hypothesis (suggested by analogy), and tested like any other scientific hypothesis. If verified, it is a new induction, and can then be included in the relevant summary induction.

Now, a summary law does not explain any of the included laws. It is only when the more comprehensive law is something more than a summary that it can be said to account for other laws which can be derived from it. Thus, for example, Newton's theory of gravitation (especially in its original causal sense) is an explanation of Kepler's three laws and of Galilei's Law of Falling Bodies; the Kinetic Theory of Gases is an explanation of Boyle's, Avogadro's and Gay-Lussac's Laws (and of the separate laws which they summarize); and the Undulatory Theory of Light explains Snell's Law of Refraction (and the laws of which this is a summary) by reducing the bending of a ray of light, as it passes from one medium to another of different density, to differences in the velocities of light in the two media.

There is a tendency to distinguish such a more

comprehensive law from the less comprehensive laws, which it explains, by calling it a *Theory*. In the preceding paragraph this distinction in nomenclature has been observed. It is a convenient distinction. The reason why these more comprehensive laws are called theories, rather than laws, may be justified as follows: in the first place, they are usually of a more speculative character than the subsidiary laws which they include, and in that sense they are more theoretical. The subsidiary or secondary laws keep more closely to the facts, and are more nearly descriptive than are the so-called theories. The theories are, in a sense, inductions from the laws, in the same way as the laws are inductions from the facts of observation. The theories are, consequently, at a further remove from the facts than are the laws; the laws may be true even if the theories are false, but the theories cannot be true if their derivative laws are false. The theories are, therefore, less probable than the laws, according to the principles of probability explained in Chapter XXII, § 2. Another reason may be this: looking at the laws objectively, that is to say, as natural laws, and not merely as verbal formulas for them, it is fairly obvious that to call the theories also laws amounts to counting the same laws twice over. The actual uniformities in the phenomena concerned may be accurately described either by reference to the secondary laws or by reference to the systematizing theory. But the two do not represent different uniformities; and, since the theories are the more speculative, and also come

after the discovery of the laws, the term "law" is naturally retained for the less comprehensive but earlier discoveries. Incidentally, the foregoing explanation may also account for the fact that we usually speak of *discovering* a law and *inventing* a theory. The word "theory" suggests at once a formula, or a formulated explanation or hypothesis, rather than an objective uniformity. But the reverse is the case with the word "law."

The invention of theories, in the sense just explained, marks an important step forward in the history of a science. For theories colligate secondary laws, just as laws colligate or order facts. Theories, therefore, mark further progress in the discovery of systematic order in the phenomena of nature. Laws of a less comprehensive character, which cannot be deduced from theories, are felt to be unexplained, to hang in the air, so to say, and are often referred to as merely *empirical* laws, in the sense, namely, that they only sum up, or just describe, the experienced or observed facts, without explaining them adequately. It should be remarked, however, that, at other times, the expression "empirical law" is confined to inductions which are based on simple enumeration, and which are, therefore, not so reliable as inductions based on the stricter inductive methods. (See Chapter XX, § 1, above.)

§ 4. *The Logical Basis of Induction.*

It has already been pointed out several times that most, if not all, generalizations are inferences from particular or individual cases or instances. And we

have already considered to some extent the question of the justification (if any) of such inferences. We have suggested that this procedure is partly, at all events, the result of a practical drive felt by mankind. The possibility of anticipating events and preparing to meet them, and such regularity or orderliness as are sufficient to provide a basis for such intelligent anticipations, are among the most urgent practical needs of life. Needs, it is true, do not always create their objects, and it would be absurd to suggest (as some people have indeed suggested) that human needs *produce* natural order or law; but such needs do prompt us to *seek* order, and to *find* it, if it is there. Hence the assumption, or rather the hope, of the Uniformity of Nature.

But we may also consider for a brief moment the *logical* (or rational, or intellectual) motive, as distinguished from the *practical* motive, that prompts generalizations from individual cases. The fundamental working idea, or assumption, in question is what I call *The Principle of the Uniformity of Reasons*, and which I formulate as follows: *Whatever is regarded as a sufficient reason in any one case must be regarded as a sufficient reason in all cases of the same type.* The force of this principle may be illustrated by a simple example. Suppose that we have secured suitable instances (one positive, and one negative instance of the precise type required) for the application of the method of difference, and we arrive at the conclusion that *d* and *z* (to use our previous symbols) are causally connected. Now strictly speaking (as has been

pointed out by hostile critics of the canons of induction) the inference warranted (if any inference be warranted at all, as these critics would say) refers only to the particular case involved, namely, in *that* case *d* was causally connected with *z*. How do we come to generalize that *d* and *z* are *always* causally connected? The answer is that, wittingly or unwittingly, our thought proceeds on the working assumption or principle that if *d* as such was really the cause or reason of *z* in one instance (which is what the method of difference is supposed to have established), then it will always be the cause or reason of *z*. It is this general assumption, applicable to all cases of generalization, that is formulated above as the Principle of the Uniformity of Reasons.

§ 5. *The Validity of Science.*

Science, like all knowledge, is based partly on observation and partly on inference, and both these processes are exposed to error. Hallucinations, and illusions, and fallacies are common enough to warn any sensible person against excessive confidence in his views. Even science, in spite of all the caution taken in its construction, is not infallible, for, in addition to the possible sources of error just indicated, there is an element of uncertainty inherent in the very character of inductive procedure, which plays an all-important rôle in science. Science usually proceeds, and rightly proceeds, on the assumption that the phenomena which we observe are probably in some sense the products of conditions operating according to laws and regularities of some kind;

and the business of science is to discover these conditions and laws. The procedure is of the nature of what is known as an *inverse* or reverse process, and may be compared, to some extent, with that of ascertaining the factors which may have produced a certain numerical or algebraic product. To such a problem there is usually more than one answer possible, and all that can be done is to enumerate all the possible answers. The case of natural phenomena is much more difficult. It is not possible, in this case, to enumerate all possible answers. The purely quantitative part of a suggested natural law may be determined with comparative certainty, but the rest of the solution is always liable to challenge by a rival solution, if not now, then later on. Still, this is no reason for scepticism. One can only test hypotheses that have actually been suggested, and embrace the one that best survives the ordeal of verification. How can one examine hypotheses not yet suggested? One can only cultivate a sufficiently open mind to pay due consideration to a better hypothesis, if and when it is put forward. Generally speaking, an hypothesis that has stood a long and severe test only calls for modification, rather than for rejection, even when it does eventually break down. So the scientific results gradually accumulated by generations of workers may well be accepted with the confidence that is placed in what is highly probable, if not with that absolute certainty which is reserved for what is beyond all doubt. The uncertainty which attaches even to scientific knowledge has prompted some people to search for a

starting-point or principle that is established beyond all doubt, and which may serve as the sure foundation of further knowledge based upon it. But this kind of Archimedean fulcrum, for the raising of science, has not yet been discovered. For the most part, observation is regarded as the safest basis of knowledge. In a wide sense the popular proverb "seeing is believing" is commonly endorsed in science as well as in everyday life. If we cannot believe what we perceive, what *shall* we believe? Scientific knowledge consists mainly of what is known from observation and what is believed to be inferable from what has been observed. Even observation, however, is not infallible. It has already been indicated that, even apart from hallucination, observation or perception usually includes elements of interpretation or explanation, which may be wrong. The line between what is called observation, on the one hand, and avowed explanation, on the other, or between so-called facts, on the one hand, and avowed theories, on the other, is not always easy to draw. The difference between them is mostly a difference of degree, rather than a difference of kind. Even the reliability of observation depends in a measure on our calling up the correct interpretative elements to blend with what is given in sense-impression; and we do not always succeed in doing so. There would thus appear to be no indubitable starting-point for knowledge. But in practical life and in science we do not worry about the apparent absence of such an immovable foundation. What actually happens may be suggested by a parable,

The ancient Indians sought for a sure foundation for the earth, and so they suggested that the earth rested on an elephant. But the elephant likewise, it was felt, needed a sure resting-place, and so it was suggested that the elephant stood on a tortoise. Now we have long since abandoned this kind of search for an elephant or a tortoise on which the earth might rest securely. We are quite content with the idea that the earth and the other planets or indeed all the stars, sustain one another gravitationally in such a manner that they can all move freely and safely in their courses, without any risk of tumbling down. So it is with our beliefs: we put our faith in the co-operation or the convergence of our observations and other judgments or beliefs. We do not suspect all observations simply because some of them have proved to be unreliable. For the most part, we believe what we observe, and we only doubt an observation when it conflicts with other experiences. If this is true already of observation, it applies with even greater force to inference, and especially to the inductive inferences by which science is more especially built up. None of the inductive methods can be applied so rigorously as to escape all cavil, and some of them are not very satisfactory at the best. The generalizations which rest on the Method of Simple Enumeration, for instance, always have a low degree of probability. The greater the number of observations on which they are based, the greater is their probability; but the probability is never very high. The other inductive methods depend for their reliability

on our ability to detect all the relevant circumstances, and to vary these as much as possible. Now, the investigator may fall short in both these respects. In ignoring certain circumstances as irrelevant he relies more or less on his previous knowledge of them, and that knowledge also is not beyond cavil. Yet, it is only by relying on previous knowledge that an investigation can be kept within manageable bounds. Similarly with the range of variability of all the relevant circumstances. The greater the range of variation of one relevant circumstance at a time, the more probable is the conclusion concerning the connection between a certain condition and a certain consequent that is based upon it, in accordance with one or other of the simpler inductive methods. But it is rarely possible to vary just one relevant circumstance at a time. How then, it may be asked, does science ever get a start? The answer is similar to that already given above with reference to observation. We rely on the harmony or mutual support of the whole of our knowledge. If the new conclusion harmonizes with the rest of our knowledge or our beliefs we accept it; if not, we sometimes reject it, and sometimes we readjust our previous beliefs in such a way that the new belief and the refashioned old beliefs should be consistent. In this way, old knowledge promotes the acquisition of new knowledge, while the new knowledge helps either to confirm or to correct old beliefs. And, as human experience grows more and more extensive, and human knowledge becomes more and more comprehensive, and embraces vast ranges of experience

colligated into self-consistent systems, which also harmonize with one another, so science gains in probability, and approximates nearer and nearer to certainty, even if it should never quite reach it.

CONCLUSION

CHAPTER XXV

SOME GENERAL PROBLEMS OF INFERENCE

THERE are certain general problems in connection with inference which may be considered briefly in this chapter.

§ 1. *The Objective Basis of Inference.*

The acts by which we draw inferences are obviously mental acts. Suppose for a moment that there were no minds capable of thinking, in that case there could be no such thing as inference. But this does not mean that inference is entirely a subjective matter, entirely the product of consciousness. Inference has an objective basis, without which it would be no more significant than the capricious play of fancy. The objective basis of inference is the actual connection between things, or events, in the world of reality. A typical kind of objective connection between events is the causal connection. And in that case it is obvious that the causes of events, when known, become the reasons for our inferences. Take away such objective causal connections, and human reasoning becomes but an idle causerie, or at most an unaccountable habit of associating ideas. Valid inference, consequently, has a two-fold basis. It presumes objective connections in the world of reality; and it assumes the possibility of our somehow apprehending these objective connections so as to use them as grounds for inferences.

This at once raises the question of the relation between the objective connections between the events in the world of reality to which our thought refers, and the logical connections between our thoughts

when we reason about those events. The usual tendency to simplify things generally expresses itself in this instance in an attempt to reduce either to the other, and to recognize one kind of nexus only. On the one hand there are the so-called idealists who endeavour to reduce the connections between material events to logical connections, and so come to regard the universe as essentially a structure of ideas (or Ideas, with a capital). On the other hand there are the materialists who endeavour to reduce the logical connections between ideas to causal connections between cerebral processes. Philosophers commonly support the former view, while men of science (if they think about these matters at all) usually favour the latter view. There are numerous exceptions, of course, in the ranks of both philosophers and scientists. There is really very little to be said in favour of either of these extreme views; and the whole problem has no special relevance for the study of Logic as such. The problem pertains to the Theory of Knowledge, or to Metaphysics generally. So far as Logic is concerned, there is no point in attempting to merge the material nexus in the logical, or the logical in the material. The two can be regarded simply as co-existent or parallel and apparently different.

Even so it remains true that the systematic character of our knowledge is based on the systematic character of reality. The more adequately we endeavour to understand anything, the farther must we pursue its ramifications in the whole system of reality, and in the end (or ideal limit) the complete knowledge of anything (even of the little "flower in the crannied wall," on which Tennyson mused) would require a knowledge of the entire system of things, which alone,

according to Spinoza, is *substance*, that is, self-supporting, both objectively and logically.

§ 2. *Inference and the Particular.*

According to some, inference is always from particular (or individual) cases to other particular cases, and when a general proposition is employed in deductive inference, the general proposition serves only as a memorandum of individual cases observed, and the inference is really from those individual cases, and not from the general proposition. On the other hand most logicians go to the other extreme and maintain that inference is never from particulars, but always from some general or universal character or aspect of the particulars in question.

The controversy is unfortunately complicated by various philosophical considerations which are of little, if any, logical importance. The main points of interest may be set out as follows.

If by " particular " is meant anything apprehended in such a way as to involve no reference to anything general (in other words, in such a way as to involve no comparison whatever with anything else, no apprehension of any of its characteristics, or features, or qualities as common to it and other things), then certainly there is nothing that we think or speak about that can be described as particular. The apprehension of such particulars, if and when it takes place, is inevitably speechless, inarticulate. At that stage there can be no such thing as the apprehension of an object, or event, and its place in any kind of objective order. At that stage, therefore, there can be no such thing as logical inference, or thought of any kind. There can only be some kind of instinctive

reaction, or practical orientation, no more. In fact, at that stage, the possibility of forming judgments, even the simplest judgments, has not yet emerged. For, as has already been explained in Chapter II, § 3, even the most rudimentary judgment requires at least one concept for predicate; and a concept is never particular in the sense now supposed. In this sense, therefore, it may be said that inference, or indeed, thought generally, is not concerned with the particular as such. That, however, is not really what is usually meant when we speak about the "particular."

If, on the other hand, the term "particular" is used in an unsophisticated sense, as synonymous with "individual" or "singular," that is, as denoting this or that member of a class or kind, then it should be obvious from the foregoing chapters that some inferences are from particulars to particulars, other inferences are from particulars to general propositions, others are from general propositions to a particular case or cases, and yet others are from general propositions to general propositions. Thus, for instance, the mediate inferences discussed in Chapter VIII, and inference from circumstantial evidence, discussed in Chapter XVI, are certainly inferences from particulars (in the sense of singular propositions, or individual cases), and so are most inductive inferences. On the other hand, deductive inference proper is inference from general propositions, so that neither of the extreme views (usually associated with the name of Mill and his opponents) is really correct. Inferences are not always from particulars nor are they always from the general, but they are sometimes from the one and sometimes from the other, and sometimes from both

What is really characteristic of all reasoning is its exploitation of connections. This characteristic is perhaps most obvious in the case of inference from circumstantial evidence, where we see very clearly, even in comparatively simple cases, the feature of interconnection between the circumstances involved, and the corresponding convergence and interlinking of the evidence which leads to the inferential construction of the complex, systematic whole. In the simpler types of inference, sometimes described as "linear" inference, the interconnection is not so marked. But the difference is only one of degree. As already remarked earlier in the book, the farther a problem or a discussion is pursued, the more ramified, or systematic, does it become. Logicians who are committed to the extreme view that inference always involves something general, or what they call a "universal," will probably insist that even inference from circumstantial evidence involves a universal, only in this case it is what they call a "concrete universal," as opposed to an "abstract universal." But to call a system a "universal" is to attempt to save a theory by obscuring the real issue.

§ 3. *The Principle of Uniformity of Reasons.*

There is a certain fundamental assumption of all inference which appears to have escaped attention except in so far as to mislead some logicians in their conception of the ultimate type of all inference. The assumption, or postulate, in question may be formulated as follows; *Whatever is regarded as a sufficient reason in any one case must be regarded as a sufficient reason in all cases of the same type.* Or, to express it negatively, *Nothing can be regarded as a sufficient reason*

in any one case unless it can also be regarded as a sufficient reason in all cases of that kind. The principle may be regarded as the logical parallel to the Principle of the Uniformity of Nature but is more comprehensive. Phenomena of the same kind exhibit the same kind of objective connections. Conclusions of the same kind must be explained in the same kind of way. At least that is so potentially, for in some cases (for instance in most cases of inference from circumstantial evidence) we are concerned with what is unique, or not likely to happen again. In any case, the principle just formulated is the counterpart or complement to the principle which lies at the basis of the warning against the fallacy known as *argumentum a dicto simpliciter ad dictum secundum quid,* and its converse. The objection to this kind of argument is that it involves a neglect of the sound maxim that "circumstances alter cases." What is true of cases of a certain general type may not be true of cases in which special circumstances are operative. Now the converse or the complementary principle to it is this: Similar cases must be treated in the same way, unless it can be shown that there are special circumstances requiring special consideration.[1] And that is virtually the Principle of the Uniformity of Reasons formulated above.

Now this principle of the uniformity of reasons is at the back of all reasoning. And every attempt made to formulate the general principle of any special type of inference is made out of deference to the principle of the uniformity of reasons. Examples of such principles we may find in the *dictum de omni et*

[1] A glimmer of this seems to be visible in some uses of the proverb "What is sauce for goose is sauce for gander."

nullo (or its substitutes) in connection with syllogisms of Fig. I, or the axiom that "things which are equal to the same thing are equal to one another," and so on. Now such general principles are really assumptions at the back of the corresponding types of argument; they are not actual premises of the arguments themselves. Failing to grasp this difference between an actual premise and a postulate, or principle, some logicians have endeavoured to show that all arguments, including inductive arguments, etc., are really syllogistic arguments, and no more. This they can only do by treating the relevant postulate as the major premise, and the whole of the actual premises as minor premise. But if this is carried through consistently then every ordinary syllogism would really be a double syllogism—one as it actually is, and another when the general principle of syllogistic inference is treated as the major premise, and the actual premises are made to function together as minor premise. And even then the principle would still be assumed. The whole attempt is extravagant. Even if much more could be said in its favour than is the case, the student of actual inference would still find it far more important and profitable to study the *differences* between the main types of inference than their alleged sameness.

A few words may be added on the relation of the principle of uniformity of reasons to the *dictum de omni et nullo*. In a sense the principle is the reverse of the *dictum*. According to the *dictum*, whatever can be asserted of a type may be asserted of any instance of it; according to the principle, whatever can be truly asserted of an instance may be asserted of the relevant type, and therefore of any other instance of that type.

The principle is clearly the more fundamental. The *dictum* gives no indication how general truths may be obtained; the principle does. The principle, moreover, meets the difficulties felt by empirical logicians like Mill, who regard the syllogism, when based on the mere *dictum*, as question-begging.

§ 4. *Concluding Remarks.*

The study of Logic has sometimes been described as a kind of refined intellectual game carried out according to certain rules. Its value as a mental discipline might have been very considerable even so. In reality, however, it is as serious a thing as life itself. Pursued in the right spirit, the study of Logic is an invaluable training in the art of self-criticism, which is so necessary to the peace and welfare of humanity. Some of the world's profoundest thinkers looked to the cultivation of Reason as the foundation of the future unity and harmony of mankind. This touching faith in the power of Reason may be unseasonable in an age notorious for its cult of the Irrational. But, if the Principle of the Uniformity of Reasons is properly understood, the justification of this faith will become clear, and so will Spinoza's view that "it is the passions that divide men, Reason brings them together." For it is in accordance with this Principle that (as Spinoza says) "men who seek their own welfare under the guidance of Reason desire nothing for themselves which they do not wish also for the rest of mankind." And it is the same principle which is at the basis of Kant's formulation of the moral law: "Act in such a way that you can will the maxim of your act to become a universal law."

APPENDIX

NOTE A.

INTENSION AND EXTENSION OF TERMS

A GENERAL concrete term, like *animal*, *house*, *triangle*, etc., refers to a class of things, and the things referred to have certain qualities or attributes in common, in virtue of which they are regarded as belonging to the same class. Now the *things* to which the term refers constitute the *extension* of that term; the *qualities* or attributes which it suggests constitute its *intension*. Again, of the qualities possessed in common by a class of things some are regarded as essential to that class, so that no object lacking any one of those qualities would be regarded as belonging to that class. What these essential qualities are is usually a matter of agreement, or convention, among experts in the relevant field of study, who enumerate these essential qualities in the definition of the term concerned. Such essential qualities as are included in the definition of a term are commonly described as its *conventional intension* or *connotation*. In so far as a term suggests qualities which do not exactly coincide with those constituting its *connotation*, we speak of them as constituting its *subjective intension*, because it may vary from individual to individual, and with the same individual from time to time. Lastly, we may wish to refer to *all* the qualities actually possessed by a class of objects, even those qualities not yet known to us, and for this purpose the expression *objective intension* or *comprehension* may be used. The term *denotation* is commonly used as synonymous with *extension*, as defined above, but it is sometimes used with special reference to a specified universe of discourse,

Generally speaking, there is a rough kind of inverse relation between connotation and denotation—as the connotation of a term is increased, so the classes of objects denoted by it tend to diminish. For example, the term "triangle" denotes all kinds of triangles; but if we add the qualification (or determinant) "equilateral," then we get the term "equilateral triangle" which connotes more, but denotes less than the term "triangle." Conversely, if we want a term to denote more classes of things than it does, then we must drop part of the connotation. Thus, for instance, if we want to refer to all three-sided figures, then we cannot use the term "equilateral triangle," but must drop the qualification "equilateral," and so get less connotation and more extension or denotation. Similarly, if we want to refer, not to English people only, but to all the people in their empire, then the term "British" must be substituted for "English," thereby reducing the connotation and enlarging the denotation. The inverse ratio, however, does not always hold good. If the qualification "equiangular" be added to the term "equilateral triangle," then the connotation is enriched, but the extension remains the same. Similarly, if "equilateral" or "equiangular" (but not both) be omitted from "equiangular equilateral triangle," it will make a change in connotation but not in denotation. Again, the connotation of the terms "man," "American," "Frenchman," etc., is unaffected by fluctuations in the relevant population.

NOTE B.

THE LOGICAL SUBJECT OF A SENTENCE

In order to restate ordinary sentences accurately in the appropriate categorical form of proposition it is necessary, first of all, to determine the logical subject of the sentence. Young students find it rather difficult to do so. They generally tend to treat the first noun of the sentence as its subject, which is not always correct; and sometimes they even pick out a noun from a nominal phrase or clause, instead of taking the whole phrase or clause, as the term. The following suggestions may help them to handle ordinary sentences more accurately.

The subject of a sentence, considered logically (as distinguished from minute grammatical analysis), may be described as the theme of the sentence—what it is about. When a context is given, there is no difficulty in determining the theme, no matter what the order of words may be. The most direct indication of the theme is given when we are given the question to which the sentence is the answer, for the question states the theme. Suppose the question put is, "Who is Mr. Roosevelt?" The answer given may be, "Mr. Roosevelt is the President of the United States of America." Obviously the subject of the sentence is "Mr. Roosevelt," and the predicate, or explanation, is "the President of the U.S.A." But, suppose the question is, "Who is the President of the U.S.A.?" The answer given may still be expressed in the same sentence as before. But now the subject and predicate are reversed. So that the mere order of terms is not

decisive. This kind of thing presents no difficulty when the context is given. Usually, however, as a matter of economy in printing exercises, the context is not given. In that case one cannot always be sure what is the subject, and what the predicate. But the student can always ask himself what question the sentence might answer, or what its context might be. By briefly stating his view, he can generally justify his logical analysis of the sentence. One or two examples may serve as illustrations of the way in which such isolated sentences should be considered. Take the sentence "Time destroys the enthusiasm of people who are immersed in routine work." Many young students would probably regard "Time" as the subject, and the rest as the predicate. But is "Time" really the theme of the sentence? Is it not more likely to be either "people who are immersed in routine work," or "the enthusiasm of people who are immersed in routine work"? In the former case, which seems preferable, the sentence should be restated thus: "All people who are immersed in routine work are people whose enthusiasm is destroyed in (or by) time." Similarly, in the sentence, "A man shows his real character when he is tempted greatly," the theme or subject is not "a man," but "the real character of a man"; and the sentence should be restated thus: "The real character of a man is revealed when he is tempted greatly." Of course, it is also possible to be too subtle in this kind of practice. For the most part sentences in ordinary use present no serious difficulty, as people do, on the whole, tend to put the subject first, and in a straightforward way. But there are exceptions, and students must avoid a merely mechanical treatment of such exercises.

NOTE C.

THE QUANTITY AND QUALITY OF SENTENCES

THE four unambiguous propositional forms are not frequently used in ordinary speech, nor is it desirable that they should be. But the student who cannot restate ordinary sentences in one or other of the forms *SaP*, *Sep*, *Sip*, *Sop*, probably does not adequately understand the meaning of those sentences. Partly the difficulty may arise from a failure to recognize the terms of the sentences. This difficulty has been dealt with in Note B. Here it is proposed to indicate some of the commoner ways in which differences of quantity and quality are expressed in ordinary sentences.

SaP. Very common substitutes for "all," "every," or "each," are the words "always," "invariably," "without exception." For example, "Equilateral triangles are always (or invariably, or without exception) equiangular" means "All equilateral triangles are equiangular." More usually, of course, "always" and "invariably" have other meanings, and the student must use his common sense. For instance, "The planets are always in motion," does not mean "All the planets are in motion," but "All the planets are always in motion." Here "always" is strictly an adverb of time, not an indication of quantity, and forms part of the predicate.

Another way of indicating *SaP* is by means of the form *S must be P*. For example, the sentence "An equilateral triangle must be equiangular" means "All equilateral triangles are equiangular," and also conveys the suggestion that this can be proved.

Still another way of expressing *SaP* is by means of a rhetorical device based on what is known as the argument *a fortiori* ("how much more so"). For instance, the sentence "Even the wisest men are fallible" means "All men are fallible," for if the wisest are fallible, how much more so the others. Similarly with "The longest lane comes to an end," and so on.

SeP. The word "never" is sometimes used for "no." For example, the statement "Right-angled triangles are never equilateral" means "No right-angled triangles are equilateral." But "never" also, and more usually, has the meaning of "at no time," and the two uses of the word must not be confused. The sentence "The planets never stop" does not mean "No planets stop," but "All the planets are bodies which *never* stop."

S cannot be P is another way of saying *SeP*. For example, "A triangle cannot have two sides together equal to the third" means "No triangle has, etc.," with the additional suggestion that this can be proved.

Still other ways of expressing universal negative propositions are by the use of "only," "alone," or "exclusively" in affirmative sentences. The fact that the sentences are affirmative in form usually occasions some difficulty to beginners. *Only S's are P*, *S's alone are P*, or *S's exclusively are P* all mean *No non-S's are P* or *No things other than S's are P*—universal negative propositions. When the student has learned the eductions he will see that *No non-S's are P* implies *All P's are S*, so that the above forms can also be restated in this way. For example, "Only British subjects are British civil servants" can be restated as "No foreign subjects are British civil servants," or as "All British civil servants are British subjects"; but

the affirmative form is not so close to the original as the negative form.

SiP. Fairly common substitutes for " some " are " sometimes," " occasionally," " frequently," etc. For instance, " Men sometimes are masters of their fate " means " Some men are masters of their fate "—fate not being a thing that the same man can master on some occasions and not on others.

Another form occasionally used to express *SiP* is *S may be P*. For example, " A triangle may be scalene " means " Some triangles are scalene." The word " may " has, of course, also other meanings, such as " to be allowed," etc.

SoP. What has just been said about " sometimes," etc., as substitutes for " some " is also relevant here. The copula, of course, will in this case be " are not " instead of " are." Similarly, " may not be " and " need not be " are sometimes used for " some are not." For example, " A triangle need not (or may not) be equilateral " means " Some triangles are not equilateral." Of course, " may not " and " need not " also have other meanings—" not allowed," and " not compelled " respectively.

There are certain other ways of expressing *SoP* which many beginners find a little difficult, because the sentences appear to be affirmative in form. The types of sentence in question are these: *Few S's are P*, *S's are seldom* (*or rarely*) *P*, or *S's are hardly ever P*. They all mean *Some* (*or most*) *S's are not P*. The difficulty with " few " is its liability to be confused with " a few." " A few books are profitable " means " Some books *are* profitable "; but " Few books are profitable " means " Most books *are not* profitable." Writers on Logic, while agreeing that the above forms (*Few S's*

are P, etc.) mean *SoP*, generally add that they imply in addition also *SiP*. But this is not correct. No doubt it happens sometimes, perhaps even frequently, that *Few S's are P*, etc., are intended to mean both *Most S's are not P* and *Some S's are P*; but not always. For example, a person who says "Few politicians are perfectly sincere" certainly means that most of them are not; but he may not mean that some of them are —he may just want to express himself with moderation. This is certainly the case when the words "if any" are added to "few," or "if ever" to "seldom" and "rarely." "Few people, if any, are always candid" just means "Most people are not always candid," and no more.

NOTE D.

THE LAW OF CONTRADICTION

The Law of Contradiction cannot be proved; it is an ultimate assumption or postulate. But it can be defended effectively. Suppose somebody says, " The Law of Contradiction is not true," then one can say to him that, since he does not acknowledge this law, there is no reason, so far as he is concerned, why the assertions that " The Law of Contradiction is true " and " The Law of Contradiction is not true " should not both of them be true. So he has no real ground for objecting to the view that the Law in question is true.

Various attempts have been made to undermine the Law of Contradiction by showing that there are cases in which a proposition may actually *imply* its own contradictory. According to one version of an ancient paradox, an Athenian is supposed to say " I am a liar." It is then argued that if the statement is true, then he is telling the truth, and is therefore not a liar; but if the statement is not true, then he is also not a liar. In either case, consequently, the proposition implies its own contradictory. This is really a feeble argument. Even a liar does not tell lies always, so that the question whether the Athenian is or is not a liar cannot be decided by the truth of the above statement, which consequently does not imply its own contradictory.

A revised version of the above story makes the Athenian say, " I am lying." Now, it is argued, if he is lying, then his assertion is true, and consequently he is not lying. If he is not lying, then his statement is a lie. In either case, therefore, the statement implies its

contradictory. This version is more subtle, but no less irrelevant. To be lying is to say something which the speaker knows to be untrue. The statement "I am lying," in other words, really means "I say . . . although I know it to be untrue." Restated in this way, it is obvious that the statement is incomplete, and really does not make sufficient sense to enable anybody to say whether the speaker is or is not lying. To turn the incomplete expression into a proposition, he would have to say something like this, "I am lying in saying that the sun is shining." Now the statement is either true or false, and may be judged by reference to the relevant facts. If the sun is not shining at the time, then the whole proposition is true (the speaker is lying), if the sun is actually shining, then the whole statement is false (the speaker is not lying); but in neither case can the assertion be said to imply its contradictory.

NOTE E.

UNIVERSE OF DISCOURSE

By " the universe of discourse " (or *suppositio*) is meant the sphere of reference to which a remark or discussion is intended to apply. The idea is an old one, but was first given prominence about the middle of the nineteenth century by Boole and De Morgan, two mathematicians who were among the founders of Symbolic Logic. " The universe of discourse," Boole wrote, " is sometimes limited to a small portion of the actual universe of things, and is sometimes co-extensive with it " (*Laws of Thought*, Ch. XI). More commonly the reference is " limited to a small portion of the actual universe," and so the expression " limited universe" is sometimes used instead of " universe of discourse." The importance of taking account of the universe of discourse lies in the fact that when, as usually happens, there is a mutual understanding about the sphere of reference, people use elliptic expressions, which, if taken literally, would be quite misleading. The same word or expression may have quite different meanings in different contexts or " universes." When discussing historical characters, " Shakespeare " will be understood to refer to the person so named; when discussing portraits, " Shakespeare " will be understood to mean " the portrait of Shakespeare "; when discussing literature, " Shakespeare " will be understood to mean " the works of Shakespeare "; and so on.

In the course of the nineteenth century, however, the term " universe of discourse," or briefly " universe " or " world," acquired a rather different meaning.

Writers referred to "worlds" of illusion, mythology, fable, sheer madness, etc., as well as to the world of real things, etc. William James, who enumerated a considerable number of "worlds," explained that "the total world . . . is composed of the realities *plus* the fancies and illusions," and that each limited world has "its own special and separate style of existence," which, however, may lapse when we cease to attend to it (*Principles of Psychology*, I, Ch. XXI). Except perhaps as a description of the psychology of make-believe, this kind of view is misleading. Take, for instance, the statement "The wrath of the Homeric gods is terrible." The "universe of discourse" is that of mythology; and works on mythology are a part of the real universe. What the above statement means is that "the Homeric poems describe the gods as terribly wrathful." The significance of the universe of discourse is to remind us that the reference is, not to real gods living in a special world of their own, but to a literary account of gods in whom former ages believed.

The universe of discourse should always be borne in mind in order to avoid mistakes, and unnecessary conundrums. In the text of the book attention has already been directed to the way in which the meaning of negative terms is determined by the "limited universe," and so on. It is important to bear in mind, when dealing with formal inference, that no statement can imply anything outside its universe of discourse. When dealing with symbols no such universe can be indicated. But symbols are not fool-proof, and symbolic manipulation must not be allowed to exonerate one from the exercise of common sense. If, for example, the subject term of a proposition denotes the whole universe of discourse, then it is wrong to infer from the

assertion anything about the contradictory term of the subject, because that involves trespassing outside the sphere of reference. Suppose, e.g., you are given the proposition "No novelist has written a completely satisfactory novel." The universe of discourse here is that of "novelists," because everybody who writes a novel is called a novelist. Now the propositional form of the sentence is *SeP*, and formally it has an inverse, namely, $\bar{S}iP$, which should mean "Some person who is not a novelist has written a completely satisfactory novel." The statement is, not untrue, but just nonsense, because all who write novels (of any kind) are called "novelists," so that no "non-novelist" writes a novel; and the inverse, $\bar{S}iP$, just contradicts itself in asserting that "Some person who has not written a novel has written a completely satisfactory novel." This comes of wandering outside the universe of discourse. (See the author's *Studies in Logic*, pp. 66 ff., for a full historical and critical account of the "universe of discourse.")

NOTE F.

EXISTENTIAL IMPORT OF CATEGORICAL PROPOSITIONS

WRITERS on Symbolic Logic have given prominence to the question whether categorical propositions, or some of them, imply the existence of objects denoted by the subject term. The problem has become unduly complicated by various other issues. An adequate discussion of the question would take up too much space. Moreover, the author has already discussed it very fully in his *Studies in Logic* (Camb. Univ. Press, 1905), which may be consulted by those interested in it. Here it will be sufficient to indicate briefly some of the main issues.

The first thing to note is that no proposition can imply the *existence* of things; at most it can only imply that the person who asserts it *believes* in the existence of what the subject denotes. It is in this sense that the question must be understood. Again, it is obvious that, for the most part, people have enough to do to think about real things, situations, etc. So that, generally speaking, most assertions may be assumed, in the absence of any indications to the contrary, to concern what the speaker or writer regards as real. Still, exceptions do occur. People do consider mere plans and possibilities, especially with regard to the future, and they do sometimes contemplate "footless fancies." Another consideration which it is important to bear in mind at the outset is the meaning of "existence" or "reality," as the whole issue has been confused by nebulous views on it. Here it will be assumed that the terms existence and reality are used

in their strict sense, and not with a variable, fanciful meaning. By a real "ship" will be meant a ship actually constructed of timber, or steel, etc.; not the mere idea or plan of a ship. Similarly, a real fairy would be one actually living; not a mere story or picture of a fairy. And so on.

Now, it is contended by some writers on Symbolic Logic that universal propositions do not imply the existence of anything denoted by the subject terms, but that particular propositions do. If this were true, then the inference of particular conclusions from universal premises would be invalid, and this would upset a good deal of the teaching of traditional formal logic. The contention, however, is unwarranted. According to it, the proposition "All equilateral triangles are equiangular" does not imply any existents; but the proposition "Some triangles are equiangular" does. Yet the objects referred to in both cases are precisely the same, because the only triangles which are equiangular are the equilateral triangles. Again, suppose the Admiralty were to announce that "Some of the cruisers for which provisions were made in the last budget are not being constructed," this could obviously not be taken to imply that the cruisers in question exist. Similarly, when the director of a building society, while discussing plans with the architect or builder, says, "Some of the houses will be rather expensive," it is obviously not implied that the said houses are already, or ever will be, built. But there is no need to multiply examples. There is as little justification for alleging that particular propositions always imply the existence of their subjects, as for alleging that universal propositions do; and the writers in question make no such claim for the latter.

When confronted with examples like the above, the answer usually given is that by "existence" they also mean "conventional existence" in some "universe" which need not be part of the real world. But this is an evasion of the issue. What is the use of contending that the ships or the houses, referred to in the above propositions, "exist in the world of plans or designs." This means that "plans of ships" or "plans of houses" exist, not the "ships" or "houses." Similarly, the contention that fairies exist in the realm of fiction means that "literary stories about fairies" exist, not "fairies." If "existence" does not mean existence, then it is rather futile to stress the existential implication of any statement.

The plain fact is that, in their attempt to express ordinary propositions in forms resembling algebraic equations, writers on Symbolic Logic have been tempted to distort the meaning of nearly all propositions. *SaP* is represented by $S\bar{P} = 0$; *SeP*, by $SP = 0$. But the assertions, "All men are mortal," "All squares are equilateral," etc., attribute certain qualities to their subjects, and are not concerned with "immortal men" or "non-equilateral squares." It is a distortion of the facts to say that they *merely* mean that these things don't exist If this kind of reduction were carried through consistently, then the statement "The rings of Saturn are real" (i.e. not mere appearances) would be construed as "No rings of Saturn that are not real are real" ($S\bar{P} = 0$). Of course, nobody would be so foolish as to put it that way; but it shows the arbitrariness of interpreting all universal propositions non-existentially.

The false step of putting a merely non-existential interpretation upon universal propositions naturally

led to the equally false step of imposing a merely existential interpretation upon particular propositions, as their contradictories. But it is all unjustifiable; and if Symbolic Logic, with its mock algebra, cannot get on without such misinterpretations, then so much the worse for it. Actually, attempts are being made to develop it along different lines. In the meantime, there is no need to worry about the validity of particular conclusions inferred from universal premises in accordance with classical Logic.

Problems of reality, or of agreement with reality, cannot be settled in a purely formal manner, but by the study of facts. As Kant and other eminent philosophers have taught long ago, existence cannot be inferred from any predications of qualities, or attributes, or relations, much less from bare forms. The " formal " consideration of a problem, even a deductive problem, is rarely more than a first approach to the real problem. Even in mathematics, applied mathematics involves a great deal more than pure mathematics. When dealing with the actual problems of life or science (as distinguished from merely formal, symbolical manipulations) content or subject-matter is at least as important as form.

NOTE G.

MODAL PROPOSITIONS

Modal propositions are propositions which, unlike ordinary categorical propositions, express not merely what is spontaneously believed to be the relation of the predicate to the subject, but also some indication of further reflection on the justification for asserting that relation. There are three types of modal propositions, namely:—

> *Apodictic*: *S must be P* (or *S cannot be P*);
> *Assertoric*: *S is actually P* (or *S is actually not P*);
> *Problematic*: *S may be P* (or *S may not be P*).

The *apodictic proposition* expresses the conviction of a necessary connection (or repugnance) between the terms S and P. For example, "Equilateral triangles must be equiangular" (or "Right-angled triangles cannot be equilateral"). This type of proposition is frequently used when the speaker feels that he can *prove* that *S is* (*or is not*) *P*, as in the cases cited. Of course, the form may be abused to deceive others, or even oneself, in an environment which demands "firm convictions." The *problematic proposition* expresses some uncertainty about the relation of P to S, when there is some evidence, but not sufficient. For example, "The sun may shine to-morrow" (or "Smith may not lose his post"). The *assertoric proposition* looks usually just like an ordinary categorical proposition, but normally consists in the re-assertion of a spontaneous (or unreflective) categorical proposition after reflection caused by some kind of challenge. Suppose somebody

asserts that "Astrologers frequently forecast future events accurately." This may be just an ordinary categorical statement. But if he is challenged with the question "But how can they?" and he replies, after reflection, "I don't know; but *actually they do*," then the categorical has become an assertoric proposition.

It should be noted that the contradictory of an apodictic proposition is a problematic proposition of opposite quality, and *vice versa*.

As has already been explained elsewhere, the apodictic form is sometimes used to express a universal categorical proposition, and the problematic form is occasionally used to express a particular categorical proposition. Conversely, a particular categorical proposition is sometimes used to express a problematic proposition (*cf.* Note F).

PREDICABLES AND CATEGORIES

THE predicables are certain terms which are commonly used to indicate the principal ways in which the predicate of an affirmative proposition may be related to its subject. The oldest known list of Predicables dates from Aristotle, but it was slightly modified by Porphyry. This modified list consists of five terms, namely, *genus, species, differentia, proprium, accidens*. They are obtained in this way. When the predicate is not essential to the subject, then it is called an "accidens" (or "accident") of it—e.g. "Some authors are wealthy." It is not essential for an author to be wealthy, but just a happy "accident." On the other hand, the predicate may express something that is regarded as essential to the subject, so that anything of which the predicate could not be affirmed would not be called by the subject-term. Now, if the predicate is essential to the subject, then there are four possible cases. (1) The predicate may denote a class of which the subject is a sub-class, e.g. "Rectangles are parallelograms," or "Carriages are conveyances." In such cases the predicate is said to be a *genus* of the subject. (2) If the subject is a singular term, and the predicate denotes a class of which it is a member, or instance, then the predicate is a *species*, e.g. "Mr. Roosevelt is an American Statesman." The same predicable is applied when the predicate is a sub-class of the subject, as, e.g. in "Some parallelograms are rectangles." (3) Again, the predicate may express some attribute of the subject which distinguishes it from other sub-classes

(or species) of the same genus. In this case the predicate is called a *differentia*—e.g. "Isosceles triangles have two equal sides." (4) Lastly, the predicate may express some essential attribute different from the preceding three, but deducible from them. It is then called a *proprium*—e.g. "Equilateral triangles are equiangular." The predicables are difficult to apply to ordinary assertions, or even to scientific propositions other than mathematical ones.

The term *accidens*, it should be noted, is also used in quite another sense, namely, in contrast with *substance*. Whatever can be regarded as existing by itself is commonly called a substance or thing; quantities, qualities, relations, etc., cannot exist by themselves but only in or between things (or substances), so they are each described as an *accidens*, a term which expresses this dependence on things. This use of the term *accidens* has arisen in connection with the doctrine of *Categories*, of which the following account is given here in order to prevent the rather common confusion between Predicables and Categories, due to the fact that both are concerned with predicates, and perhaps also to the use of the term "predicament" for category. *Category* is commonly used as synonymous with *class*; but that is not the sense in which it is employed in philosophy. Here it means *an ultimate kind of predicate*; hence the other name sometimes used instead of it, *predicaments* (or *praedicamenta*). The subject was first discussed by Aristotle, who drew up a list of ten such categories, namely, *Substance* (or *Thing*), *Quantity*, *Quality*, *Relation*, *Place*, *Time*, *Position*, *State*, *Activity*, and *Passivity* (i.e. being acted on). These categories were regarded by Aristotle as expressing *ultimate modes of being* as well as *ultimate modes of apprehension* (or

predication). "Modes of being" must not be confused with "things"; to be a "thing" (or "substance") is only one mode of being; and all other modes of being may exist in one "substance." For example, one may predicate of Aristotle that he was a man (substance), who lived in Athens (place), in the fourth century B.C. (time), taught philosophy (activity), was accused of atheism (passivity), felt alarmed (state), but had friends (relation), and so on. The categories were conceived as *ultimate*, in the sense that none of them was regarded as reducible to any other category, whereas any derivative predicate could be reduced to one or other of the categories. The predicate "blue," for instance, can be explained as a "colour," and "colour" as a "quality"; but "quality" cannot be explained by reference to any other term. Similarly with the other categories.

Various other attempts have been made to draw up lists of categories, but none of them is regarded as satisfactory. The best known of them is that proposed by the philosopher Immanuel Kant (1724–1804). Kant did not believe that we can have any knowledge of reality as it is in itself, but only of its appearances to us. The ultimate kinds of predicates, therefore, express, not ultimate "modes of being," but only ultimate "modes of apprehension," or the most general ways or forms in which we apprehend reality Space (Aristotle's "Place") and Time were regarded by Kant as two such forms of apprehension. He did not call them categories, because he arbitrarily restricted this term to *conceptual* forms, whereas Space and Time are *perceptual* forms. Spaces and times are, respectively, only *parts* of one all-inclusive Space and one all-inclusive Time, whereas the categories express *kinds*,

Forms of Propositions.	*Corresponding Categories.*
I. Quantity:	I. Quantity:
(*a*) Singular (*This S is P*).	(*a*) Unity.
(*b*) Particular (*Some S's are P*).	(*b*) Plurality.
(*c*) Universal (*All S's are P*).	(*c*) Totality.
II. Quality:	II. Quality:
(*a*) Affirmative (*S is P*).	(*a*) Reality.
(*b*) Negative (*S is not P*).	(*b*) Negation.
(*c*) Infinite (*S is non-P*).	(*c*) Limitation.
III. Relation:	III. Relation:
(*a*) Categorical (*S is P*).	(*a*) Substance and Quality.
(*b*) Hypothetical (*If A, then C*).	(*b*) Cause and Effect.
(*c*) Disjunctive (*Either A or B*).	(*c*) Reciprocity (or Activity and Passivity).
IV. Modality:	IV. Modality:
(*a*) Problematic (*S may be P*).	(*a*) Possibility and Impossibility.
(*b*) Assertoric (*S is P*).	(*b*) Existence and Non-existence.
(*c*) Apodictic (*S must be P*).	(*c*) Necessity and Contingency.

of which the relevant objects are *instances* or members. (In a list of ultimate kinds of predicates this distinction is of no great importance.) Kant's list of categories (in his use of the term) is interesting to students of Logic because of the way in which Kant derived it. Kant had great faith in traditional Logic, and he assumed that, since thought is always expressed in propositions, the main types of proposition, as classified in books on Logic, must correspond to the ultimate types of conceptual predication (i.e. categories, in his sense). But his pedantic love of symmetry induced him to doctor the traditional classification of propositions to a slight extent so as to have three sub-classes under each principal heading. His list of propositional forms, or judgments, and the corresponding categories, is given in the accompanying Table, from which it will be seen that, if due allowance is made for Space and Time, Kant's list of ultimate kinds of predicates is not so very different from that of Aristotle.

NOTE I.

SYMBOLIC LOGIC

THE term "Symbolic Logic" is now used for a whole group of studies variously known as "Algebra of Logic," "Mathematical Logic," "Logistic," etc. Practically all the work in this field has been done by mathematicians, whose aims and methods differed considerably. In spite of its long history, it cannot be said even now that the subject has crystallized into a sufficiently coherent and sound body of doctrine suitable for beginners in Logic. An attempt to give an intelligible and satisfactory account of the subject would require more space than has been devoted in this textbook to the whole of Formal Logic; and indeed there are several much more lengthy accounts of Symbolic Logic which have not succeeded in presenting the subject satisfactorily. It may be advisable to say just a little about Symbolic Logic, because reference has been made to it in previous Notes, when repudiating some of the attacks on certain doctrines of the traditional Formal Logic. But the sole object of this Note is to give a general idea of what Symbolic Logic is about. Anybody who is sufficiently interested to seek fuller information about the group of studies in question is advised to consult *An Introduction to Symbolic Logic*, by S. K. Langer (1937).

The triumph of abstract reasoning in the field of pure mathematics has, throughout the ages, tempted people to extend the range of such *a priori* or deductive thinking to other fields. When applied to the explanation of physical and biological facts this kind of

"rationalism" proved barren; and the pioneers of modern science had to stress the importance of observing things, instead of merely spinning intellectual yarns. Mathematics, however, played so important a rôle in the new advances of physical science that the old faith in the powers of abstract thought maintained itself. Descartes' successful and fruitful application of algebraic methods to the study of geometry naturally enhanced the reputation of algebraic methods, and the idea occurred to several people that the entire field of general, formal, or deductive reasoning might be fruitfully studied by an extension of algebraic methods. This seemed all the more plausible inasmuch as thinkers like Thomas Hobbes and others regarded all reasoning as a kind of reckoning. Accordingly, various attempts were made, in the seventeenth century, to construct a kind of generalized art or science of reckoning, so as to extend the field of Formal Logic. The most important worker in this field at that time was Leibniz, the eminent mathematician and philosopher. He made three attempts in this direction, but did not succeed in producing anything satisfactory. To apply algebraic methods to another branch of mathematics is one thing, to extend their range of application beyond mathematics was a much more formidable task.

During the eighteenth century the subject fell into abeyance. Kant, the greatest philosopher of the period, was not impressed by the attempts to construct a kind of Algebraic Logic; and the respect which he showed for the traditional Formal Logic tended to discourage any tinkering with it. So nothing much was done in this field until about the middle of the nineteenth century.

Augustus de Morgan (1806–78), an English mathema-

tician, in his *Formal Logic* (1847) and *Syllabus of a Proposed System of Logic* (1860), etc., attempted to extend the sphere of Formal Logic, and especially of the doctrine of the syllogism, by taking into account other kinds of relations between the terms of a proposition than those commonly expressed by the copula "is" (or "are"). He pointed out that the copula in any case has several different meanings, for it expresses sometimes the identity of objects having different names, or simply the identity of certain objects, or, again, that one concept has the same content as another (*cf.* p. 51). What is important in all these relations is that they are *convertible* and *transitive*, for it is on these properties that inference depends. But there are other kinds of relations having these properties, so that other types of inference can be based upon them. De Morgan, accordingly, laid the foundations of what is now commonly known as a Logic of Relations. Some of these types of inference have already been dealt with on pp. 76 f., 83 ff., and need not be considered here again. In the subsequent treatment of relations the points mainly considered are the *number of terms related*, the *symmetry* of the relation, and its *transitiveness*. The terms may be two, three, or more, though the number may sometimes be arbitrary. For instance, in "Restaurants feed the public," the relation is *dyadic*; in "Restaurants provide food for the public," it is *triadic*; in "Restaurants provide food for the benefit of the public," it is *tetradic*. A *symmetric* relation is one that is convertible, e.g. if "A is partner of B," then "B is a partner of A"; otherwise it is called *asymmetric*, e.g. if "A is older than B," then B is *not* older than A, and so on. The nature of transitive relations has already been explained on

pp. 83 f. It should be obvious that the actual character of a relationship must be determined in each case by a knowledge of the relevant facts. Symbols can only be used to express such a knowledge, not as a substitute for it.

The first moderately successful attempt to construct an algebraic, or quasi-algebraic, Logic (or a " Symbolic Logic," as it was called subsequently) was made by George Boole (1815–64), another English mathematician. He published two important contributions to the subject, namely, *The Mathematical Analysis of Logic, Being an Essay Towards a Calculus of Deductive Reasoning* (1847), and *An Investigation of the Laws of Thought on which are founded the Mathematical Theories of Logic and Probability* (1854). Boole employed the literal symbols x, y, z, etc., to represent " things as subjects of our conceptions "; the signs of operation $+$, $-$, $\times$, for the combination or resolution of conceptions of things " so as to form new conceptions involving the same elements "; and the sign $=$ for identity (or the equivalent of " is "). Thus x may stand for man, y for mortal. The plus sign may mean " and," also " or "; for if the class " men " and the class " women " are combined (say, $x + z$), then the new class consists of " men *and* women," but you describe any member of the combined class as " a man *or* woman." The multiplication sign ($\times$) was used to indicate the common members of the classes on which it operated, e.g. if x represents " black objects " and z represents " cows," then xz will represent " black cows." Naturally xx, or x^2, must always $= x$ in the " Algebra of Logic," for if we select from the class " cows " those that are " cows " (zz) the result is just " cows " (z). The subtraction sign ($-$) he used as equivalent to " except,"

so that $z - x$ could stand for "all cows except those which are black." He used the numeral 1 to indicate the whole "universe of discourse," and 0 to indicate that a given class (x or y, etc.) is empty. Thus $1 - x$ stood for what is now usually expressed by $\bar{x}$. And he expressed the Law of Contradiction in the equation $x(1 - x) = 0$. To indicate an undistributed term the symbol v was used, *All x's are y's* was represented by $x = vy$, whereas *All x's are all y's* was expressed by $x = y$. Boole's methods were too complicated to be widely acceptable, but he stimulated many other mathematicians to continue his attempt to construct a comprehensive calculus of deductive inference.

During the later decades of the nineteenth century, E. Schroeder (1841–1902), C. S. Peirce (1839–1914), and several others made further attempts to develop an Algebra of Logic. In his *Symbolic Logic* (1881, 2nd ed. 1894), John Venn tried to criticize and systematize all the results achieved up to his time. This appears to have been the first time that the name "Symbolic Logic" was used. Venn used the term "class" in the sense of a "compartment," which might be empty (what was later called a "null class"), and he aimed at the elaboration of a "class calculus," in which all propositions were treated as asserting the "existence" (in some "universe" or other) or "non-existence" of certain classes denoted by their terms (or the occupation or emptiness of the "compartments"). Venn represented the four familiar types of categorical proposition as follows:—(1) *All x's are y* by $x\bar{y} = 0$; (2) *No x's are y* by $xy = 0$; (3) *Some x's are y* by $xy > 0$; (4) *Some x's are not y* by $x\bar{y} > 0$ (where "> 0" means "different from, or more than, nothing"). Having once adopted the non-existential interpretation of the

universal propositions, it was inevitable that Venn should adopt the existential interpretation of the particular propositions as the contradictories of the universals. Venn was aware that he was doing something arbitrary; but he regarded it as a legitimate convention for the purpose of his calculus· and his conception of "classes" as compartments (or mere pigeon-holes), and of "conventional" modes of existence helped to salve his conscience.

The four traditional types of categorical proposition, however, were not sufficient for a calculus. For, even with two terms, say x and y, and the two values 0 and 1 (as interpreted by Boole) there are sixteen conceivable propositions, namely:—

$$\begin{array}{llll} xy = 0 & xy > 0 & xy = 1 & xy < 1 \\ x\bar{y} = 0 & x\bar{y} > 0 & x\bar{y} = 1 & x\bar{y} < 1 \\ \bar{x}y = 0 & \bar{x}y > 0 & \bar{x}y = 1 & \bar{x}y < 1 \\ \bar{x}\bar{y} = 0 & \bar{x}\bar{y} > 0 & \bar{x}\bar{y} = 1 & \bar{x}\bar{y} < 1 \end{array}$$

(where < 1 means "does not constitute the whole universe of discourse").

Now, with only two relevant terms, say x and y, the entire universe of discourse must be constituted by the four combinations xy, $x\bar{y}$, $\bar{x}y$, $\bar{x}\bar{y}$, so that $xy + x\bar{y} + \bar{x}y + \bar{x}\bar{y} = 1$. This renders possible certain substitutions for each of the above sixteen propositional forms Instead of $xy = 0$, we can say $x\bar{y} + \bar{x}y + \bar{x}\bar{y} = 1$. Similarly, $xy = 1$ can be replaced by $x\bar{y} + \bar{x}y + \bar{x}\bar{y} = 0$; $xy > 0$, by $x\bar{y} + \bar{x}y + \bar{x}\bar{y} < 1$; and $xy < 1$, by $x\bar{y} + \bar{x}y + \bar{x}\bar{y} > 0$; and so on.

Venn also attempted to adapt the class calculus to a propositional calculus, in which x, y, z, etc., each represents the truth of a proposition; $\bar{x}$, etc., its falsity; 1 "the sum-total of possibilities"; and 0 "no possi-

bility." The formula $xy + x\bar{y} + \bar{x}y + \bar{x}\bar{y} = 1$ would then mean that "the truth of both x and y, the truth of x only, the truth of y only, and the falsity of both x and y between them exhaust all the possibilities."

The consideration of the meaning of $x\bar{y} = 0$ led Venn to give an unorthodox meaning to the term "implication." The meaning of $x\bar{y} = 0$ is that "All x's are y"; and, when x and y stand for two propositions, it means that it is impossible for x to be true when y is false, in other words, that x *implies* y, or that "if x is true, then y is true." But now it is possible that $y = 1$, in other words, that y constitutes the entire universe of discourse, or exhausts all possibilities, so that y is independent of x. Venn, accordingly, concluded that "the phrase 'x implies y' does not imply that the facts concerned [when x and y in $x\bar{y} = 0$ represent classes] are known to be connected, or that the one proposition is formally inferrible from the other." This use of "implication" has now been widely adopted, and is sometimes described as non-formal or "material" implication.

The most important contributions towards Symbolic Logic during the past sixty years or so, have also all been made by mathematicians. Their chief aim, however, has been rather different. They did not seek to develop a universal calculus, but to provide a perfectly sound foundation for mathematics. This branch of Symbolic Logic is commonly described as Logistic. The chief workers in this field are, or have been, R. Dedekind, G. Cantor, G. Frege, G. Peano, B. Russell, and A. N. Whitehead. The subject is, of course, of great importance for mathematics, but hardly concerns students of elementary Logic, though some of the teachers of Logic are among the most vociferous camp-followers of the men at arms.

One of the recent innovations in the study of Symbolic Logic consists in an attempt to turn it all into a calculus of propositions. The older writers on Symbolic Logic either confined themselves to a calculus of classes or tried to develop a calculus of propositions, alongside of a calculus of classes. Now, however, the tendency is to replace the calculus of classes by a calculus of propositions, on the ground that any relation between classes is naturally expressed in a proposition. If successful, this kind of method would simplify the processes of Symbolic Logic. Some people, however, still believe that it is better to begin with a calculus of classes.

The symbols now used in Symbolic Logic are much more numerous than those used formerly, and some of the old ones have had their meaning changed. A few examples must suffice here by way of illustration. In the class calculus the symbol " $<$ " now means " is included in," so that $a < b$ means " All a's are b," or " The class a is included in class b "; but when a represents an individual and b a class, then " a is b " is expressed by $a \epsilon b$. The differentiation is due to the fact that the relation of an instance to its class is different from that of a species to its genus, the latter being always a transitive relation, whereas the former is not. It is customary to use the earlier letters of the alphabet (a, b, c, etc.) to represent classes, and the later letters (p, q, r, etc.) for propositions. The sign for the contradictory of a proposition has also been differentiated from that of a class—thus $\bar{a}'$ means class non-a, but p′ means the contradictory of the proposition p. A similar differentiation has been introduced as regards the symbols for implication, the sign $\supset$ is used to indicate that one proposition implies

another. Thus "$p \supset q$" means "p implies q," but $a < b$ means "class a is included in, or implies, b." The sign for "or" is also different in the case of propositions from what it is in the case of classes: $a + b$, for classes; but $p \vee q$ for propositions. The sign for logical multiplication is also different in the two cases: $a \times b$, or simply ab, for classes, but $p.q$ for propositions (p *dot* q means "both p and q are true"). Lastly, the numeral 1 still represents the "universe of discourse," or the class which contains all classes under consideration; but in the case of proposition it means "true," whereas 0 still means "false," in the case of propositions.

Many or most of the early workers in the field of Symbolic Logic dreamed of a potent calculus which would make it possible to go through elaborate processes of reasoning, and arrive at accurate results, more or less mechanically, without thinking of the significance of the intermediate stages, after the manner of mathematics. This idea has now been abandoned by most mathematicians in favour of a strictly correct development of mathematics. It is well to bear in mind that, outside the realm of pure mathematics, sound reasoning always requires close attention to the relevant facts, and cannot be assured by mere conformity to highly abstract symbolic processes. Excessive devotion to the manipulation of symbols sometimes tends to undermine common sense, and to lead to the confusion of an instrument of truth with truth itself.

The way in which the method of following mere symbolism to what is euphemistically called "its logical conclusion" sometimes leads to solemn nonsense may be illustrated by reference to a few logistic results, which are sometimes uttered with bated breath as though they were profound mysteries.

(1) In Algebra $x = x + 0$, which is commonly paraphrased as meaning that a given quantity remains the same if " nothing " is added to it. The logistician assumes that this must have its analogue in his scheme. Consequently, since x represents a class, and 0 a " null class " (i.e. an empty class or compartment), it is concluded that " any class *includes* the null class, $0 < x$," whatever this may mean. Actually what the algebraic formula means is that " if no addition is made to a quantity (this is obviously what is meant by " adding ' nothing ' "), then the quantity remains the same." The analogous logistic meaning should be that " if no addition is made to a class, the class remains the same." There is no justification for referring to this as a " paradox."

The alleged " paradox " can easily be capped by another; and it is surprising that this had not been noticed before. Here it is. (2) In Algebra $x = x - 0$. On the above logistic analogy, this should mean that " any class *excludes* the null class." Putting the two " paradoxes " together, it would appear that every class both includes and excludes the null class; and some logisticians may easily see here additional evidence for repudiating the Law of Contradiction (see Note D, near the end). Of course, as a matter of common sense, what $x = x - 0$ really means is that " if no subtraction is made from a quantity, or a class, then the quantity, or the class, remains the same," and there is nothing " paradoxical " or mysterious about it.

(3) By misapplying the same sort of ingenuity to symbols representing propositions (instead of classes), and making use of the above misinterpretation of $x = x + 0$, some logisticians solemnly pronounce the new mystery that " a false proposition implies any

other proposition." Even allowing for the fact that "implies" (as has already been explained above with reference to Venn), in Logistic, does not always mean what it says, this "paradox of implication" sounds like solemn nonsense. The argument runs as follows. Since the "null class" is included in any class, we can express this in the form of $0 < x$. Now, in the case of propositions 0 means "false," and the falsity of p is expressed by p'. Hence the propositional equivalent of $0 < x$ is $p' \supset q$, that is, any false proposition implies any other proposition! As we have already shown that $0 < x$ is a misinterpretation of $x = x + 0$, it is unnecessary to waste time over its propositional analogue. The utmost that the alleged "paradox" can mean is that "the falsity of a proposition is consistent with any number of other propositions being true or false." Is this really worth stating? There is much more sense and relevancy in the statement of traditional Logic that the falsity of a proposition implies the truth of its contradictory.

(4) Another logistical pronouncement is to the effect that "any proposition implies a true proposition." This high-sounding conclusion is obtained in the following way. Assuming the formulae $0 < x$, as already explained, and $x < 1$ (i.e. that any class x is included in the universe of discourse), and giving them the analogous propositional interpretation, the following propositional formulae are obtained. First, as explained in (3), $p' \supset q$, i.e. "a false proposition implies any other proposition." Secondly, $p \supset q$, that is, "a true proposition implies any other true proposition." By putting the two together the result obtained is that "a true proposition is implied by any true or false proposition," or, in other words, "any proposition

implies a true proposition." Now, we have already rejected the assumed interpretation of $o < x$, and its alleged propositional meaning, and so we reject the novel pronouncement based upon them. Moreover, all that this "paradox" could mean is that "the truth or falsity of any one proposition is consistent with the truth of any other proposition." This statement is as insignificant as it is sweeping. And, here again, traditional Logic is much more to the point in insisting that a true proposition must be consistent with other true propositions, and that any propositions inconsistent with true propositions must be false.

The above conundrums have been described by logisticians as "paradoxes" in the sense of "surprises." For some reason or other some writers seem more anxious to spring new surprises than to discover new truths. Indeed, some of their attacks on the existing Logic seem to be part of this game of springing surprises. But this kind of thing can easily over-reach itself, and one should not perhaps be altogether surprised at the supreme "paradox" of certain logisticians to the effect that the whole problem of logically validated knowledge is mainly, if not wholly, a matter of language!

NOTE J.

FALLACIES

THE best way of dealing with Fallacies, that is, invalid arguments, is to point out where precisely they violate some definite logical principle or principles. To discuss all possible fallacies would involve going over the whole ground of Logic again in order to indicate and name the different ways of straying from correct procedure. This seems quite unnecessary. But it may be of some use to enumerate and explain the more familiar designations of the chief types of wrong thinking, as these names are frequently used in season and out of season.

The oldest known account of Fallacies was intended as a warning against certain tricks of specious argumentation. Aristotle, accordingly, only enumerated fifteen types, which he arranged in two groups, namely (I) those which consist in a misleading use of language (*in dictione*), and (II) those which are not merely verbal (*extra dictionem*). Group I contained six types, Group II nine types, as follows.

I. Verbal Fallacies.

(*a*) *Equivocation*, or the ambiguous use of words or terms. For example, it is not uncommon for a man to describe his opponent's criticisms as "impertinent," ostensively in the sense of "not pertinent," i.e. irrelevant, but with the inuendo that they are "impudent." Similarly, the word "all" may be used in the course of the same argument sometimes in the sense of "each" and sometimes with the meaning of "all together."

Practically every word is ambiguous, but as a rule the context makes it clear which of its meanings is intended.

(*b*) *Amphibology* (or *Amphiboly*), or ambiguity in the construction of a sentence. For instance, in the statement "Smith only promised a donation," the word "only" might refer to "Smith," or to "promised," or to "a donation" (as distinguished from an annual subscription).

(*c*) *Composition*, or using a term collectively (that is, for "all together") when it should be used distributively (that is, for "one at a time"). For example, the sentence "Members of Parliament exercise little influence on foreign policy" may be true of the individual members, and not of all the members as a body, that is, of the whole Parliament.

(*d*) *Division* is the converse of the preceding type of fallacy, and consists in the use of a term distributively when it should be used collectively. For instance, the statement "Englishmen are fond of sport," may be true of the English people as a body, and not of each individual Englishman.

(*e*) *Accent*, or distorting the meaning of a sentence by wrongly emphasizing some part of it. For example, in the assertion "Jones thinks that Smith will succeed," if emphasis is laid on *Jones* the suggestion may be conveyed that nobody else thinks so, and if emphasis is laid on *thinks*, there will be an insinuation that the opinion is not held with much conviction.

(*f*) *Figure of Speech* (*figura dictionis*). This does not mean here a metaphor, but the misinterpretation of an expression by supposing that its prefix, or suffix, has the same meaning as it has in other cases. For example, the terms "insufficient," "inaccurate," "inattentive," etc., mean, respectively, "not sufficient," "not accu-

rate," "not attentive," etc.; "invaluable," however, does not mean "not valuable" or "worthless," but "very valuable" (so valuable that its value cannot be assessed—"in-valuable" in this sense). Similarly, "manageable," "breakable," etc., respectively mean "capable of being managed," "capable of being broken," etc.; but "readable" means "good to read" (of a book) as well as "legible" (of handwriting); and "desirable" usually means "what *ought to be* desired" or "what *it is right* to desire."

II. Non-Verbal Fallacies.

(*a*) *Accidens* or *Accident.* This is the confusion of the accidental with the essential. For example, one may know a person quite well, in spite of failing to recognize him when he is in disguise, or dressed, or made up, in some unusual way. (*cf.* Note H.)

(*b*) *A dicto simpliciter ad dictum secundum quid,* or arguing from a general rule to a special case, without allowing for the special circumstances. For example, generally it is a crime to kill anybody; but the legal execution of criminals, and the destruction of enemy soldiers in wartime, present special circumstances, and "circumstances alter cases." Similarly, it is generally right for people to be allowed freedom of speech. But can it be rightly claimed by those who want to exploit this right in order to establish a régime which would deprive all others of such, and even more basic, rights?

(*c*) *A dicto secundum quid ad dictum simpliciter.* This is the converse of the preceding type of fallacy, and consists in arguing from what holds good in special circumstances to a general rule in which those circumstances are ignored. For example, the fact that certain things (e.g. the destruction of enemy life and property,

etc.) are allowed in the special circumstances of warfare, is no ground for allowing them in peace time.

(*d*) *Petitio Principii* or *Begging the Question.* This fallacy consists in assuming a "principle," that is, a general proposition, which, if true, would justify the desired conclusion, but which one has no right to assume. For example, it is sometimes argued that it is wrong to tax tea and beer, because that is to tax the food of the working classes. The tacit or explicit assumption made is that "all taxes on the food of the working classes is wrong"; but the assumption needs adequate justification, and some definition of its limits.

Reference may be made to J. S. Mill's criticism of the syllogism as committing the fallacy of *petitio principii* (*System of Logic*, II, Ch. III). He argued that in the syllogism *MaP*, *SaM*, ∴ *SaP*, the major premise *MaP* is unwarranted if it is not known already that the *S*'s (which are all *M*) *are all P* (*SaP*). So that the truth of the conclusion, so far from being inferrible from the major premise, is actually required in support of the major premise, which would be unwarranted without it. The criticism assumes that the major premise (*MaP*) is always obtained by an enumeration of all the relevant instances. But that is a mistake. Inductive generalizations are usually obtained from an examination of a few suitable instances, and are then applied to innumerable new instances. Moreover, many major premises consist of laws or regulations prescribed by legislative bodies. It would be absurd to suggest that the relevant cases have all been dealt with before the laws or regulations have been enacted. Obviously the laws or rules are laid down before they are applied to relevant cases.

(*e*) *Circulus in Demonstrando* or *Arguing in a Circle.*

This fallacy consists in using an unproved premise to prove a conclusion, which is then used to justify the premise. For example, if one contends that china is better than earthenware, because it is made of a better kind of clay, and, when asked why the clay is better, he argues that it is better because it is used to make china, whereas the inferior clay is used to make earthenware, then he is arguing in a circle, as just defined. China is better than earthenware because it is made of better clay; and the clay is better, because china is better than earthenware.

(*f*) *Consequens* or *Consequent.* This fallacy consists in assuming that a proposition of the form *If S is M, it is P* always implies the proposition *If S is P, it is M.* But that is not the case. See p. 122.

(*g*) *Many Questions* or *Plures Interrogationes.* This is a sophistical catch, which insists on a simple " Yes " or " No," in answer to a question that is not simple, but includes an unwarranted assumption, which cannot be touched by a plain " yes " or " no." For example, suppose the question is, " Are you still neglecting your work? " then even the answer " No " will leave the impression that you were formerly in the habit of neglecting your work. If that is not the case, then another question should have been put first, namely, " Did you ever neglect your work? " The putting of the above question tacitly assumes that this question had already been answered in the affirmative. Hence the name " many questions."

(*h*) *Non causa pro causa* or giving a bad reason. *Causa* here means *reason* (as in *because*), not *cause.* It is also sometimes called *Non sequitur*, or " inconsequence." This fallacy is essentially the same as the next.

(i) *Ignoratio Elenchi* or *Irrelevant Argument.* This fallacy arises from overlooking or ignoring what it is that has to be proved, or the real point at issue. For example, if a candidate claims that he is entitled to a University Degree, it is not relevant for him to plead that he has been working under difficult conditions; he must show that he has acquired the requisite qualifications. The varieties of irrelevancy are too many for exhaustive enumeration. But some of them have acquired sufficient notoriety to have special names assigned to them. These may be mentioned, and briefly explained here.

(1) *Argumentum ad hominem* consists in abusing the opponent instead of reasoning with him. It is illustrated in the legal story: "No case. Abuse the plaintiff's attorney."

(2) *Argumentum ad populum* consists in appealing to the passions or sentiments of the audience, instead of rationally justifying the policy urged. It is a favourite method of demagogues.

(3) *Argumentum ad baculum* consists in using the big stick against an opponent, or putting the pistol to his head, instead of reasoning with him. This kind of " argument " is greatly favoured by dictators and gangsters.

(4) *Argumentum ad verecundiam* consists in shaming people into submission, on the ground of the fame or authority of those who already accept the view advocated, instead of offering adequate evidence in support of it.

(5) *Argumentum ad ignorantiam* consists in exploiting the ignorance of the audience, and telling them mere fairy tales, or even less reputable fictions.

As has already been stated above, the original discussion of the foregoing list of fallacies was mainly intended as a warning against sophistical talkers. With the growing interest in the study of Logic, and the formulation of the various rules of correct thinking, attention was naturally directed to all violations of these rules, any such violation constituting a fallacy. In the circumstances, a new grouping of fallacies suggested itself, namely, into those which come within the province of Formal Logic, and those which come within the province of Inductive Logic. For the sake of brevity these may be named respectively Formal Fallacies, and Material or Inductive Fallacies. But there is no point in going over the whole ground again. We shall, accordingly, confine our attention here to those fallacies which have been given special names, and are not included in the above list.

A. Formal Fallacies.

(*a*) *Illicit Conversion* consists in converting *SaP* into *PaS*, or *SoP* into *PoS*. See p. 67.

(*b*) *Four Terms*, or having four terms in a syllogism instead of three. This fallacy usually results from the employment of an ambiguous middle term, which gives the appearance of a three-term syllogism, when there are really four distinct terms and no syllogism. For example:

" No gentleman makes impertinent criticisms;
" A. B. has made impertinent criticisms;
" Therefore A. B. is not a gentleman."

But if " impertinent " means " impudent " or " offensive" in the major premise, and "irrelevant" in the minor premise, then there are four terms, and no real middle term.

(*c*) *Undistributed Middle*, or the fallacy of using a syllogism in which the middle term is not distributed. See p. 89.

(*d*) *Illicit Major*, or the fallacy of drawing a negative conclusion in a syllogism (and so distributing the major term) when the major term is undistributed in the major premise. See p. 92.

(*e*) *Illicit Minor*, or the fallacy of drawing a universal conclusion in a syllogism (and so distributing the minor term) when the minor term is undistributed in the minor premise. See p. 92.

(*f*) *Affirming the Consequent* is the fallacy of positing (or accepting), in the minor premise of a mixed hypothetical syllogism, the consequent of the hypothetical major premise, and then positing, in the conclusion, the antecedent of the major premise. Thus, *If S is M it is P; S is P; therefore S is M*. It is essentially the same fallacy as that of *Consequens*, explained above. See also p. 122.

B. Material or Inductive Fallacies.

(*a*) *Non-observation*, the failure to observe, or to take into account, some fact that is relevant to the investigation, as e.g. when investigating an outbreak of typhoid, it is not observed that the water-supply is contaminated, or that some employé connected with the water-works is a typhoid carrier.

(*b*) *Mal-observation*, or seeing things wrongly, mistaking one thing for another, as e.g. when a doctor mistakes the symptoms of one illness for the symptoms of another.

(*c*) *Post hoc ergo propter hoc* ("after this, therefore because of it"), mistaking mere sequence for causal

connection, without satisfying the methods of induction. See p. 210.

(*d*) *Cum hoc ergo propter hoc* (" with this, therefore because of it "), mistaking the concurrence of certain events or conditions for causal connections, without a satisfactory application of inductive methods. See p. 210.

(*e*) *Non ceteris paribus* (" not similar in other respects ") is the fallacy of applying the Method of Difference to positive and negative instances which are not similar in other respects. See pp. 209 f.

EXERCISES

Exercises on Chapter I

a. Give a simple account of the scope of Logic.

b. What is the difference between *immediate* and *derived* judgments? Give three examples of each.

c. Explain the relation between the following terms: *belief, knowledge, mere belief*. Give two examples of each.

d. Explain the difference between a *true judgment* and a *valid inference*. Illustrate your answer, and state with which of the two Logic is concerned, and why.

e What is meant by the *formalism* of Logic?

f. What is meant by *science* and *scientific attitude*? Examine the question whether History is a science.

g. Describe briefly the two principal types of reasoning.

h. What is meant by a *scientific method*? Distinguish between the *technical* and the *logical* methods of science.

i. For what educational purpose or purposes is Logic studied?

j. Describe the chief divisions of Logic.

Exercises on Chapter II

a. What is meant by a *proposition*? How is it related to *belief* and *thought*?

b. What is meant by the *implication* of propositions, and how is it related to the problem of *inference*?

c. Describe the way in which *thought* serves as an aid to human orientation.

d. What is meant by the *terms* of a judgment (or

proposition), and what is the difference between the *subject* and the *predicate*?

e. Explain the form and meaning of *impersonal propositions*. Is there such a thing as an impersonal *judgment*? Give reasons for your view.

Exercises on Chapter III

a. What is meant by the *form* of a proposition? Illustrate your answer.

b. What is the character of a *categorical* proposition? Give examples.

c. What is meant by the *quality* of a categorical proposition, and what are its varieties? Give examples.

d. Explain carefully, and with the aid of suitable illustrations, in what sense the difference between affirmative and negative propositions is fundamental, and in what sense it is mainly a matter of convenience.

e. Why is a *genuine question* not a proposition?

f. Some questions are only *rhetorical* questions: they are not intended to *seek* but to *convey* definite opinions, and are therefore propositions. Explain, and give examples.

g. What is meant by the *quantity* of categorical propositions, and what are its varieties? Give examples.

h. Explain and illustrate the difference between a *singular* proposition and a *general* proposition.

i. Distinguish between a *universal* proposition and a *particular* proposition, and show in what way *particular* propositions are *indefinite* or indeterminate.

j. Draw up a Table of Categorical Propositions, and show how the four forms A, E, I, O are obtained.

k. Express the following sentences in the appropriate forms of categorical propositions, adding the symbolic form in each case. The first eight sentences are worked out by way of illustration.

(1) What man has done man can do.

[*All things that man has done are things that man can do. SaP.*]

(2) Plays cannot be judged by merely reading them.

[*No plays are things that can be judged by mere reading. SeP.*]

(3) Most words are infected with vagueness.

[*Some words are vague. SiP.*]

(4) What is not considered proper is not always wrong.

[*Some things not considered proper are not wrong. SoP.*]

(5) What mortal has no cause for regret?

[*All mortals are beings who have some cause for regret. SaP.*]

(6) Very few cultured people, if any, are brutal.

[*Some cultured people are not brutal. SoP.*]

(7) In the last reckoning only character really matters.

[*All that matters in the last reckoning is character. SaP.*]

(8) Even the longest lived man is not immortal.

[*No man is immortal. SeP.*]

(9) All concerted action involves subordination.

(10) People who do not cultivate the art of kindly speech can only scold.

(11) An unhappy household is not a good nursery.

(12) The properly trained humar being is never superfluous.
(13) Many erroneous opinions are held with strong conviction.
(14) Books on history sometimes throw more light on their authors than on their subject matter.
(15) Where there is no vision the people perish.
(16) Those who prepare themselves for responsible positions rarely fail to get them
(17) Few (if any) self-centred people make a success of life.
(18) Members of large families get on in the world.
(19) Man is betrayed by what is false in him.
(20) Even the right to govern ourselves badly is worth fighting for.
(21) Only men accustomed to look at all sides of a question can be scientific.
(22) Who does not feel annoyed when his prejudices are challenged?
(23) Very few couples (if any) can live in a single room without quarrelling.
(24) One cannot put old heads on young shoulders.
(25) People who live in communities are more sociable than those who live in small families.
(26) What do they know of England who only England know?
(27) That's a bad sort of education that makes folks unreasonable.
(28) "No good is certain, but the steadfast mind, The undivided will to seek the good."
(29) Grief, when there is no bitterness in it, is soothed by time.

(30) What makes and maintains a State is the idea of a common good.

(31) The eye only sees what the mind can recognize.

(32) What people want is seldom what would satisfy them if they got it.

(33) Time wears out the enthusiasm which is not converted into action.

(34) What a thing is when its growth is completed, that is its nature.

(35) The maintenance of justice is the supreme end of the law.

(36) The greatest pleasures which men can have in this world are those of discovering new truths and shaking off old prejudices.

(37) The strongest memory is no consolation for a weak intelligence.

(38) Medical students are only crammed into stupidity when they are not taught scientific method.

(39) It is the lot of a specialist to learn more and more about less and less.

(40) The problems of philosophy are the problems which the sciences ignore.

(41) In intellectual morality the fundamental virtue is patience.

(42) Those who will only study what they consider useful generally end by being of no use for anything.

(43) People who do not shirk the discipline of special training do not put their trust in mere common sense.

(44) It only makes people lazy and incompetent if you do everything for them.

(45) Even the strongest convictions, if unproved, are not reliable.

(46) More people are influenced by rhetoric than by logic.

(47) What is not permanent may yet be real.

(48) Ideas exercise an influence even when they are erroneous.

(49) He who does not increase his knowledge decreases it.

(50) Even those who know that they cannot complete their task are not free to desist from it.

l. What is meant by a *distributed term*? Describe the distribution of terms in the four types of categorical propositions.

m. Formulate and explain the general rule of formal inference relating to the distribution of terms.

EXERCISES ON CHAPTER IV

a. What is meant by *immediate inference* and what are its main kinds?

b. Formulate the *Law of Contradiction* and give an illustration of its application.

c. Formulate the *Law of Excluded Middle* and give an example of its application.

d. What is meant by the *opposition of propositions*?

e. Give and explain the *Square of Opposition*.

f. If it is assumed that *All planets move in elliptic orbits*, what other propositions (having the same terms) are (i) true, (ii) false, (iii) uncertain?

g. Supposing that *No equilateral triangles are right-angled*, what other propositions (having the same terms) are (i) true, (ii) false, (iii) uncertain?

h. Assuming that *Some hard workers are unsuccessful*, what other propositions are (i) true, (ii) false, (iii) uncertain?

i. If it is supposed that *Some worthy people are not popular*, what other propositions are (i) true, (ii) false, (iii) uncertain?

j. Answer question *f* on the assumption that the given assertion is false.

k. Answer question *g* on the assumption that the given assertion is false.

l. Answer question *h* on the assumption that the given assertion is false.

m. Answer question *i* on the assumption that the given assertion is false.

n. State the contradictory and (where possible) the contrary of the sentences given under *k* in the exercises on Chapter III.

Exercises on Chapter V

a. What are *contradictory terms*, and how are they symbolized? Give examples.

b. Given the symbols P and $\bar{P}$, is it possible to tell which of them stands for a positive term and which for a negative term? Justify your answer.

c. What is meant by the *obverse* of a proposition?

d. Show by what law of thought an obverse is obtained from each type of categorical proposition.

e. Give the obverse of each of the sentences given under *k* in the exercises on Chapter III. The first four of them are worked out here by way of illustration.

(1) The logical form is *SaP*. The obverse of this is $Se\bar{P}$. Hence, *No things that man has done are things that man cannot do.*

(2) The logical form is *SeP*. The obverse of this is $Sa\bar{P}$. Hence, *All plays are things that cannot be judged by mere reading.*

(3) The logical form is *SiP*. The obverse of this is $So\bar{P}$. Hence, *Some words are not precise in their meaning* [or *Some words are not other than vague*].

(4) The logical form is *SoP*. The obverse of this is $Si\bar{P}$. Hence, *Some things not considered proper are proper.*

f. What is meant by the *converse* of a proposition?

g. Explain how the converse is obtained from the propositional forms *SeP*, *SiP*, *SaP*.

h. Why has *SoP* no converse?

i. Give the converse, if possible, of the sentences given under *k* in the exercises on Chapter III. The first four of them are dealt with here by way of illustration.

(1) The logical form is *SaP*. The converse of this is *PiS*. Hence, *Some things that man can do are what man has done.*

(2) The logical form is *SeP*. The converse of this is *PeS*. Hence, *No things that can be judged by mere reading are plays.*

(3) The logical form is *SiP*. The converse of this is *PiS*. Hence, *Some vague things* [or *things infected with vagueness*] *are words.*

(4) The logical form is *SoP*, which has no converse.

EXERCISES ON CHAPTER VI

a. What is meant by the *contrapositive* of a proposition?

b. State the contrapositive, if possible, of the sentences given under *k* in the exercises on Chapter III. The first four of them are dealt with here as examples.

(1) The logical form is *SaP*. Its contrapositive is $\bar{P}eS$. Hence, *No thing that man cannot do is a thing that man has done.*

(2) The logical form is *SeP*. Its contrapositive is $\bar{P}iS$. Hence, *Some things that cannot be judged by mere reading are plays.*

(3) The logical form is *SiP*, which has no contrapositive.

(4) The logical form is *SoP*. Its contrapositive is $\bar{P}iS$. Hence, *Some proper things are things not considered proper.*

c. What is meant by the *inverse* of a proposition?

d. Give the inverse, where possible, of the sentences given under *k* in the exercises to Chapter III. The first four of them are worked out here as illustrations.

(1) The logical form is *SaP*. Its inverse is $\bar{S}oP$ Hence, *Some things that man has not done are not things that man can do.*

(2) The logical form is *SeP*. Its inverse is $\bar{S}iP$. Hence, *Some things other than plays [are things that] can be judged by mere reading*

(3) The logical form is *SiP*. This has no inverse.

(4) The logical form is *SoP*, which has no inverse.

e. Show the exact form of the following argument:—

Life means action. So when we cease to do, we cease to live.

f. Explain the precise form of the false conclusion referred to in the following passage:—

We all like to think that what we are interested in is important. From this we are apt to conclude falsely that what we are not interested in is unimportant.

g. Show the mutual relation of the two main assertions in the following lines:—

"At last he fell ill, as old chronicles tell,
And then, as folks said, he was not very well."

Exercises on Chapter VII

a. What is meant by *material contraries*?

b. Give the material contraries of the following statements:—

(1) Platinum is the most precious metal.

(2) Honesty is the best policy.

(3) The discovery of a new truth is a most stimulating experience.

(4) Prejudice is the worst enemy of science.

(5) Greed is the chief hindrance to honesty.

(6) The quickest way is the shortest.

c. Give the formal and material contraries of the following:—

(1) All conventions are useful.

(2) Profound books are always obscure.

(3) Masterful people are difficult to get on with.

(4) All capable people are successful.

(5) Everyone who is imprudent is unhappy.
(6) Whatever is just is expedient.

d. How exactly are the material contraries of the sentences in the preceding question related to the formal contraries?

e. What is meant by *material contradictories*?

f. Give the formal and material contradictories of the sentences in question *c*, and explain their relation to one another.

g. Why does the statement *Squares are rectangles* imply the proposition *Squares are parallelograms*, whereas the statement *Squares are not triangles* does not imply that *Squares are not plane rectilinear figures*?

h. Why does the sentence *Circles are not rectilinear figures* imply that *Circles are not quadrilaterals*, whereas the statement *Squares are quadrilaterals* does imply that *Squares are rectilinear figures*?

i. What is meant by *immediate inference by converse relation*?

j. Illustrate the difference between the *correlative* of a proposition and its *converse*.

k. Give the *correlative* and the *converse* of the following:—

(1) Good articles are always cheaper than bad ones.
(2) Horse carriages are older than motor cars.
(3) Life is more than mere existence.
(4) Knowledge is better than choice gold.
(5) Mars is much nearer the earth than is Uranus.

l Explain *immediate inference by complication of terms*, and give examples.

EXERCISES ON CHAPTER VIII

a. What is meant by *mediate inference*?

b. Name and describe the three propositions which constitute a mediate inference.

c. Why do two negative premises warrant no conclusion?

d. Why must the conclusion be affirmative when both premises are affirmative?

e. Why must the conclusion be negative if one premise is negative?

f. Why must there be a negative premise if we want to draw a negative conclusion?

g. Why must the points referred to in the two preceding questions (*e*, *f*) be proved separately? Does not either imply the other?

h. Point out the *conclusion*, the *premises*, and the *middle term* in each of the following arguments:—

(1) What is most necessary for life costs nothing. For air is most necessary for life, and it costs nothing.

(2) The earth's nearest luminary shines by reflected light. For the moon shines by reflected light, and it is nearest to the earth.

(3) Knowledge is power, because knowledge is foresight, and foresight is power.

(4) To have wants is to have hopes, and to have hopes is to be on the way of progress. To have wants is, therefore, to be on the way of progress.

(5) The surest way to keep friends is to take a real interest in them. For the surest way to keep them is to appear interested in

them, and *the* way to appear interested in them is to be really interested in them.

(6) The moon is not self-luminous, yet sheds much light. So a luminary need not be self-luminous.

i. Explain the character of *transitive relations*, and give examples.

j. What is meant by *dovetail relations*? Give examples.

k State what conclusion you would draw in each of the following cases, and give your reasons:—

(1) The line AB is equal to BC; and AC is equal to BC.

(2) The area of ABC is greater than the area of ABD, and the area of ABD is greater than that of EFG.

(3) Lady A is the sister of Dame B, and Mrs. C is the sister of Dame B.

(4) X is less intelligent than Y, and Y than Z.

(5) Lord A is the brother-in-law of Dame B, and Miss B is the daughter of Dame B.

(6) A is due south of B, and C is due east of A.

(7) The secure riveting of the plates of ships, tanks, etc., depends on the honesty of the riveters; and the lives of many men hang on such secure riveting.

(8) A is a co-director of B, and B of C.

(9) B is to the right of A, and so is C.

(10) S is a cousin of M, and so is P.

EXERCISES ON CHAPTER IX

a Explain the terms *major premise* and *minor premise*.

b. Why must the middle term be distributed once at least?

c. Which terms must be distributed in the premises to warrant (i) a *universal* conclusion, (ii) a *negative* conclusion? Give reasons.

d. Point out the *minor premise* and the *major premise* in each of the arguments given under *h* in the exercises on Chapter VIII.

e. Show that unless one premise is universal no valid conclusion can be inferred.

f. Prove that unless both premises are universal only a particular conclusion (if any) can be drawn.

g. Show that no valid conclusion can be inferred when the major premise is particular unless the minor premise is affirmative.

h. Explain, by reference to the general rules of mediate inference, why the following combinations of premises (in which the first symbol represents the major premise, the second the minor premise) cannot warrant a conclusion: EE, EO, OE, OO, II, IO, OI, IE.

Exercises on Chapter X

a. Explain the terms *syllogism* and *deduction*, and their relation to *mediate inference*.

b. Give examples of arguments which are (i) both deductive and syllogistic, (ii) deductive but not syllogistic, (iii) syllogistic but not deductive.

c. What is meant by the *figure* of a syllogism, and what are the different figures?

d. What is meant by the *mood* of a syllogism, and what is the relation between the moods and the figures of the syllogism?

e. Show how to ascertain what combination of premises is likely to warrant a conclusion in some figure or other, without determining which.

f. Show how to determine the valid moods of Figure II.

g. Show how to determine the valid moods of Figure IV.

h. Explain the derivation of the *Special Rules of Figure* I.

i. Explain the derivation of the *Special Rules of Figure* III.

j. Why cannot an O proposition be used as a premise in Figures I and IV?

k. Re-state the following arguments in such a way as to make their logical forms clear. State the major premise first, then the minor premise, then the conclusion; add the symbolic scheme at the side; and if you consider the conclusion invalid, say so, and give reasons. The first three arguments are dealt with here by way of examples.

(1) Karl Marx died unsatisfied. For he was a philosopher, and philosophers always die unsatisfied.

[All philosophers die unsatisfied, *MaP*
Karl Marx was a philospher, *SaM*
∴ Karl Marx died unsatisfied. ∴ *SaP*]

(2) Only enterprising people get on. So Jones did not get on, for he was not enterprising.

[All who get on are enterprising, *PaM*
Jones was not enterprising, *SeM*
∴ Jones did not get on. ∴ *SeP*]

(3) Those who achieve great ends are happy.

Hence young people cannot be happy, for great ends cannot be realized in youth.

[All who achieve great ends are happy, *MaP*

No young people achieve great ends, *SeM*

∴ No young people are happy. ∴ *SeP*

Fallacious conclusion. It distributes the major term, although this is not distributed in the major premise.]

(4) Smith is an optimist, and therefore patient, as all optimists are.

(5) Jones is incapable of making anybody happy. He has no capacity for enjoyment, and such people never can make anybody happy.

(6) Most poets live in a world of dreams, and therefore ignore the evils of reality, as do all people who live in a dream world.

(7) Relativity and the quantum theory baffle common sense, and so are repellent to common people, as are all new ideas that baffle common sense.

(8) Some people who have all the duties of British subjects have not got all their privileges. For naturalized British subjects incur all the duties of their status, but are not admitted to the privileges of civil service appointments.

(9) Agreeable companions do not ask questions or make criticisms. Hence animals are agreeable companions, for they do neither.

(10) This metal cannot be silver. For it is not soluble in nitric acid, as silver is.

(11)–(16) Deal in the same way with the arguments given under *h* in the exercises on Chapter VIII.

(17)–(26) Deal in the same way with the arguments (when completed) given under *k* in the exercises on Chapter VIII.

(27) How do you distinguish between *qualitative* and *quantitative deduction*?

(28) Give three examples of quantitative deduction.

EXERCISES ON CHAPTER XI

a. What is an *enthymeme*, and what different kinds of enthymemes are there? Give examples.

b. What is meant by *universe of discourse*, and how does it affect the way in which we express our thoughts?

c. Describe the precise form of each of the following arguments, supplying any proposition that may be implied without being expressed. The first two arguments are dealt with here by way of example. Propositions implied but not expressed in the given argument are put in round brackets.

(1) No belief is worth dying for, since none is certain.

[(Nothing uncertain is worth dying for,) (*MeP*)
All beliefs are uncertain, *SaM*
∴ No belief is worth dying for. ∴ *SeP*
Enthymeme of the first order. The minor premise is obverted.]

(2) Few men know their own business, for it

cannot be understood without a knowledge of things outside it.

[Those who do not know things outside it do not know their own business, *MeP*

(Most men are such as do not know things outside their own business,) (*SiM*)

∴ Most men do not know their own business. ∴ *SoP*

Enthymeme of the second order. Impersonal form of the given minor premise changed slightly to fit the context.]

(3) In his poor condition Lazarus could pay no fees, and so could get no physician.

(4) The world is round, so we can ride to the other side.

(5) The right to think what seems probable and to say what one thinks is worth dying for, as is everything on which human progress depends.

(6) James is not self-satisfied. No thinker ever is.

(7) Smith is physically weak, and therefore lacking in courage.

(8) Sheffield is an industrial city, and industrial cities are not health resorts.

(9) Jones is a poor candidate, and poor candidates always grumble about the examination papers.

(10) "White fellow works, not black fellow; black fellow gentleman."

d. What kind of enthymemes are the arguments given under *k* in the exercises on Chapter VIII?

e. Re-state arguments (1) to (5) given under *k* in the exercises on Chapter X as enthymemes of the first order.

f. Re-state arguments (6) to (10) given under *k* in the exercises on Chapter X as enthymemes of the second order.

g. Explain the terms *polysyllogism*, *prosyllogism*, *episyllogism*, and give an example of each.

h. Explain the terms *sorites* and *epicheirema*, and give an example of each.

i. What is meant by *linear inference*? Give an example.

j. Re-state the following arguments in such a way as to display their logical form, supplying (in brackets) any propositions which may be implied though not stated. Add the symbolic scheme in each case. The first argument is dealt with here as an example.

(1) The knowledge of several languages is of great educational value. For it shows us that the same word may represent different ideas, and that the same idea may be expressed by different words, and so helps us to discriminate between words and ideas, which is of great importance to clear thinking.

[Whatever shows that the same word may express different ideas, etc., helps one to discriminate between words and ideas;	*MaP*
The knowledge of several languages shows this;	*SaM*
(∴ The knowledge of several lan-	

guages helps one to discriminate between words and ideas.) (∴ *SaP*)

What helps one to discriminate between words and ideas is of great importance to clear thinking; *PaR*

(∴ The knowledge of several languages is of great importance to clear thinking.) (∴ *SaR*)

(What is of great importance to clear thinking is of great educational value,) (*RaT*)

∴ The knowledge of several languages is of great educational value. ∴ *SaT*

A sorites consisting of three enthymemes, all in Figure I.]

(2) Reserve is restraint, and restraint is painful, and pain is intolerable to the self-indulgent.

(3) Free Trade is a great boon to the working man; for it increases trade, and this cheapens articles of ordinary consumption; this gives a greater purchasing power to money, which is equivalent to a rise in real wages, and any rise in real wages is a boon to the working man.

(4) The best way to make friends is to appear to be interested in them; the best way to appear to be interested is to be really interested in them; and to be interested in others one must cease to be self-centred.

(5) The Majority Report of the Divorce Commission is a dangerous document. For it

recommends an increase in facilities for divorce. This would have the effect of loosening the marriage bond, and so weaken family life, on which the stability of the State depends.

(6) The absence of complete candour from theological discussions is justifiable. For it is a kind of compromise. Now no one disputes the necessity of compromise in politics. Why, then, should one question in the intellectual domain what is so convenient, even necessary, in the political domain?

(7) Political freedom is indispensable for a people's cultural development. For despotism lowers the nation's self-respect and checks enterprise in its most important sphere, the management of the commonwealth; and what is hostile to enterprise and self-respect is hostile to science and art.

(8) The average man regards new ideas inconsistent with his old beliefs as an evil. For the due consideration of such ideas requires a mental rearrangement—which is a laborious process. But the average man is intellectually lazy, and condemns as an evil whatever he finds disagreeable.

(9) "War begets Poverty—Poverty, Peace—
Peace begets Riches—Fate will not cease—
Riches beget Pride—Pride is War's ground—
War begets Poverty—and so the world goes round."

EXERCISES ON CHAPTER XII

a. Explain the general character of *hypothetical propositions*, with examples.

b. Express the statement *A implies B* as an hypothetical proposition, and explain its meaning fully.

c. Express the statement *A is incompatible with B* as an hypothetical proposition, and explain its meaning fully.

d. Apply the doctrine of opposition to hypothetical propositions.

e. Construct a *pure hypothetical syllogism* in each of the four Figures.

f. Explain, with examples, the principal kinds of *mixed hypothetical syllogisms*.

g. Re-state the following arguments so as to show their precise forms, supply (in round brackets) any missing propositions, and add the symbolic scheme. The first three arguments are worked out as examples.

(1) When we really want a thing we look for it with special keenness, and so we detect it sooner than do others, as happens always when people look for a thing with special keenness.

[If we look for a thing with special keenness, then we see it sooner than do others,	*If B, then C*
If we really want a thing we look for it with special keenness,	*If A, then B*

∴ If we really want a thing, we detect it sooner than do others. ∴ *If A, then C*
A pure hypothetical syllogism, Figure I, AAA.]

(2) If wishes were horses, beggars would ride; but they don't.
[If wishes were horses, beggars would ride, *If A, then C*
Beggars do not ride, *not C*
(∴ Wishes are not horses.) (∴ *not A*)
Destructive mixed hypothetical enthymeme of the third order.]

(3) If unemployment doles are given indiscriminately, there is a shortage of labour. Wages will consequently be high, and so raise the cost of production. Commodities must therefore become more expensive, and so raise the cost of living.
[If there is a shortage of labour, wages are high; *If B, then C*
If unemployment doles are given indiscriminately, there is a shortage of labour; *If A, then B*
(∴ If unemployment doles are given indiscriminately, wages are high.) (∴ *If A, then C*)
If wages are high, the cost of production rises; *If C, then D*
(∴ If unemployment doles are given indiscriminately, the cost of production rises.) (∴ *If A, then D*)

If the cost of production rises, commodities are more expensive;	*If D, then F*
(∴ If unemployment doles, etc., commodities are more expensive.)	(∴ *If A, then F*)
If commodities are more expensive, the cost of living rises,	*If F, then G*
∴ If unemployment doles, etc., the cost of living rises.	∴ *If A, then G*

A pure hypothetical sorites. All syllogisms in Figure I.]

(4) If we had not the pleasures of hope, life would lose much of its interest; but if the future could be foreseen clearly, there would be no pleasures of hope, and life would, therefore, be less interesting.

(5) If a society is to be happy, it must be productive. For in order to be happy, it must be wealthy; and if it is to be wealthy, it must be productive.

(6) If professional education is accessible to all classes, then professional fees are low. For in that case there is keen professional competition; and if there is such competition, then professional fees are low.

(7) Matters are not left entirely to simple folk anywhere. If they were, there would be no social changes anywhere; but such changes are to be met with everywhere.

(8) What can be stated in offensive terms is not to be rejected merely on that account.

For if we were to reject every assertion that can be stated in offensive terms, we should not affirm anything; yet we do affirm lots of things.

(9) If the object of marriage were bliss, then its disagreeableness to either party would be a sufficient reason for dissolving it; but it is not. The object of marriage is consequently not bliss.

(10) If God exists, then the trials of earthly life can be borne with equanimity. For if God exists, He is perfect. If He is perfect, then the human soul is immortal. And if we are immortal, then our earthly life is a small matter.

(11) A favourable state of the exchanges will lead to importation of gold; this will cause a corresponding issue of bank-notes which will occasion an advance in prices; which again will check exportation and encourage importation, tending to turn the exchanges against us.

(12) Freedom of speech is one of the conditions of the progress of civilization. For the progress of civilization depends on our deliberate adaptation of our habits and institutions to changing conditions. But such adaptation requires the correction of old errors and the acquisition of new knowledge. And these are only possible when there is complete freedom of speech.

(13) Has any small island ever had so momentous a history as Heligoland? But for it there

would have been no German Navy. But for the German Navy there would have been no Great War. But for the Great War the fate of many millions of men and women would have been very different.

(14) Unless the juridical equality of all nations is made a fundamental principle of the League of Nations the peace of the world will remain a dream. For without such equality there will be a sense of injustice. This means discontent, and discontent does not make for peace.

(15) If the law were really impartial and punished blasphemy because it offends the feelings of believers, then it ought also to punish such preaching as offends the feelings of unbelievers. But the law imposes no restraints on the believer, however offensive his teaching may be to those who do not agree with him.

EXERCISES ON CHAPTER XIII

a. Explain the general character of *alternative* (or *disjunctive*) *propositions*, and give examples.

b. Express the meaning of the statement *A implies B* as an alternative proposition, and explain its meaning.

c. Express the meaning of the statement *A is incompatible with B* in an alternative proposition, and justify the re-statement.

d. Express the substance of the following sentences in (i) *hypothetical* and (ii) *alternative propositions*:—

(1) Things which are not worth doing well should not be done at all.
(2) Candidates for the British Civil Service must be British born.
(3) Right-angled triangles are never equilateral.
(4) Parallelograms always have their opposite angles equal.
(5) One never masters a subject that does not interest him.
(6) Who ever loved that loved not at first sight?

e. Discuss the relation between the first and second proposition in each of the following pairs:—
(1) If X, then Y; either Y or not X.
(2) Either X or Y; neither X nor Y.
(3) Both X and Y; neither X nor Y.
(4) Both X and Y; either not X or not Y.
(5) Either X or Y; either not X or not Y.

f. Explain the nature of *pure disjunction syllogisms*, and give an example.

g Describe the character of *mixed disjunctive syllogisms*, and give an example.

Exercises on Chapter XIV

a. What is meant by a *dilemma*? Explain the expression "on the horns of a dilemma."

b. Explain and illustrate the difference between a *constructive* and a *destructive dilemma*.

c. Explain and illustrate the difference between a *complex* and a *simple dilemma*.

d. What is the chief fault to which dilemmas are liable? Give an example.

e. What is meant by the *rebuttal of a dilemma*, and what is its value? Give an example.

f. Re-state the following arguments so as to display their forms; supply (in round brackets) any missing propositions, and add the symbolic scheme of each argument. The first argument is dealt with here as an example.

(1) For Canada to build one or two warships in order to protect her coast would be sheer waste of money. For, except the United States, no Power could evade or overcome the British Navy; whilst if the United States resolved to invade Canada no navy in the world could prevent it.

[If the U.S. resolved to invade Canada, Canadian warships would be useless; if any other Power resolved to do so, Canadian warships would be unnecessary — *If A, then B; if C, then D;*

(But the (feared) invader must be either the U.S. or some other Power) — *(Either A or C;)*

∴ Canadian warships would be either useless or unnecessary. — *Either B or D.*

(Expenditure on what is either useless or unnecessary is sheer waste of money) — *(MaP;)*

(Expenditure on Canadian warships is expenditure on what is unnecessary or useless) — *(SaM;)*

∴ Expenditure on Canadian warships is sheer waste of money — *SaP.*

The argument consists of an abridged com-

plex constructive dilemma, followed by an abridged categorical syllogism.]

(2) Apollo behaved badly towards Orestes. For if it was wrong for Orestes to kill his mother, then Apollo should not have commanded him to do so; and if it was right, then he should have protected him from the Furies. But Apollo did neither.

(3) No honest lawyer will plead for an accused person. For the accused is either guilty or innocent. If he is guilty he ought not to be defended; and if he is innocent it must be apparent to his judges, so that he need not be defended.

(4) Those who are advocating an increase in import tariffs will be pleased in any case; for they will be pleased if their policy becomes law; and if it does not they will still be pleased, because they will continue to buy things more cheaply than they could if the tariffs were increased.

(5) To growl at human ills is a waste of time and energy.

For every ill beneath the sun
There is some remedy, or none.
Should there be one, resolve to find it;
If not, submit, and never mind it.

Additional Exercises on Chapters V–XIV

Describe the form of the arguments expressed, or referred to, in the following passages. Supply missing propositions (in round brackets) where necessary. Add the symbolic scheme of each argument. And explain

any fallacy that may be committed, or imputed, in any of them. The first six passages are dealt with here as examples.

1. Bentham was the victim of a common delusion. If a system will work, then the minutest details can be exhibited. Therefore, it is inferred, an exhibition of minute details proves that it will work.

[The delusion consists in assuming that the proposition *If A, then C* implies the proposition *If C, then A*. This kind of erroneous reasoning is known as the *fallacy of affirming the consequent*.]

2. There is not a single branch of education that when considered by itself can with truth be said to be indispensable. Can we therefore resist the conclusion that a man may dispense with education altogether?

[Here it is erroneously suggested that what is true of each one of a group of things must be true of all taken together. This kind of error is known as the *fallacy of composition*. The reverse error of supposing that what is true of all together is true of each separately is called the *fallacy of division*.]

3. By faith we may believe that those who are known to be dead are not dead. For faith is a means of believing that which we know not to be true.

[This argument commits the *fallacy of ambiguous construction* or *amphiboly*. The second sentence is only acceptable if "we know not to be true" means "we do not know to be true," but not if (as happens here) it is also taken to mean "we know to be untrue."]

4. The College staff must have been increased, for there are two new members on the Professorial Council, and all members of the Professorial Council are members of the staff.

[*Fallacious inference by added determinants.* To be *new as a member of the Council* is not the same as to be *new as a member of the staff*—an old member of the staff may be newly added to the Council.]

5. Wind and weather are unpredictable, although they are governed by laws.

[Wind and weather are unpredictable. *MaP*
Wind and weather are governed by laws *MaS*
(∴ Some things governed by laws are unpredictable). (∴ *SiP*)
Enthymeme of third order.]

6. This author is certainly confused. If I understand his book rightly, he is confused in his thinking, and if I do not understand it, then he is confused in his writing.

[*If A, then B; if C, then B.*
(Either A or C.)
∴ B in any case.
This is a *simple constructive dilemma* with the minor premise omitted.]

7. Although the magnificent person is liberal, it does not follow that the liberal person is magnificent.

8. We speak of a person as *a bad doctor* or *a bad actor*, although we should not call him *bad* in an absolute sense.

9. If it is false that "no treaties can prevent war," it must be true that "war is preventable"; but if it is true that "all treaties fail to prevent war," it does *not* follow that "war is unpreventable."

10. "Many's the time I've asked Josh what politics is, and all he can say is, 'It's what women can't understand.' There must be a power of politics in the world, for there's many things I can't understand."

11. Freedom of speech is of the essence of democracy. To renounce freedom of speech is to renounce democracy.

12. Human life will at some time disappear from the earth, for all men must die.

13. The House of Lords has two new members, because there are two new dukes, and all dukes are members of the House of Lords.

14. An import duty on sugar is beneficial to sugar-refiners, an import duty on corn is beneficial to corn-growers, an import duty on silk goods is beneficial to silk-weavers, and so forth. Now practically every member of the community is connected with some business or other, therefore a universal system of protective duties would benefit the entire community.

15. X would rather be good than bad, but he would rather be rich than good. What conclusion can be drawn?

16. The uncritical acceptance of the views of one's teachers is no real education. It fails to develop the power of independent judgment, which is the essence of real education.

17. Devout people are not necessarily moral people. Henry III of France, e.g., was a notorious seeker after forbidden things, yet he was devout, ordering prayers for the success of projected murders, and attending mass during their commission.

18. All metals, it is true, are conductors of electricity; but then the atmosphere is not a metal, and therefore cannot be a conductor of electricity.

19. The radical is not always unselfish; your mere malcontent, for example, is often rather a selfish being, and every malcontent is, of course, a radical.

20. Emperors may be good men, and good men

may be Emperors, for Marcus Aurelius was both a good man and an Emperor.

21. The rings of Saturn must be material bodies, for they are visible, and only material bodies are visible.

22. The fixed stars must be subject to the law of gravitation, because they are material bodies, and no material body is not subject to the law of gravitation.

23. Without slavery of some kind there can be no civilization. For there can be no civilization without leisure; and slavery makes leisure possible.

24. To be wealthy is not to be healthy; not to be healthy is to be miserable; therefore, to be wealthy is to be miserable.

25. Whosoever loveth wine shall not be trusted of any man, for he cannot keep a secret.

26. The line A B is equal to the line C D, for they are radii of the same circle.

27. Most of the electors were in favour of female suffrage, and most of them were Conservatives. Therefore some Conservatives were in favour of female suffrage.

28. Examiners who are excessively tender with weak candidates are unjust towards the better candidates. For they reduce the standard of the examination, and therefore the value of passing it, even for those who have reached the higher standard.

29. Excessive discipline makes one weak-willed. For undue subordination to others leaves one insufficient opportunity for the exercise and development of one's own will.

30. Power pleases the violent and proud, wealth delights the placid and timorous; youth, therefore, flies at the power, and age grovels after the riches.

31. Dogmatic teaching does not make thinking pupils, for it does not encourage independent thinking.

32. People accustomed to despotic rule find self-government very difficult when they attain to freedom, because they have not had sufficient opportunity for the exercise and development of their powers.

33. If ye only love them that love you, what merit have ye? Do not even the publicans do the same?

34. Darwin must have been very unhappy. For he said that he would feel very happy if he had only to observe and not to write; and we know, of course, that he wrote many books.

35. Free Trade must bring prosperity; for England is the richest country in the world; and this is just what you would expect if Free Trade brought prosperity.

36. If all the absurd theories of lawyers and divines were to vitiate the objects in which they are conversant, we should have no law and no religion left in the world.

37. The Helvetii, if they went through the country of the Sequani, were sure to meet with various difficulties; and if they went through the Roman province, they were exposed to the danger of opposition from Cæsar; but they were obliged to go one way or the other; therefore they were either sure of meeting with various difficulties or exposed to the danger of opposition from Cæsar.

38. Light is seen on portions of the moon which are not directly illuminated by the sun. This light must be due to the moon's own light, or to light reflected from the earth. But certain phenomena connected with eclipses show that the moon is not self-luminous. The light in question must consequently be due to reflected earth-light.

39. Why fear death? To fear death is to fear either

being deprived of all feeling or being subjected to some other kind of feeling. But if we are deprived of all feeling we shall have no evil to fear; if we are to find new kinds of sensations our existence will indeed be different, but still we shall continue to exist.

40. I am walking with a friend in the garden, and we see a moth alight upon a flower. He exclaims: "What a beautiful butterfly!" Whereupon I remark: "That is not a butterfly; it is a moth." If he asks me how I know that, the answer is: "Because butterflies, when they alight, close their wings vertically; moths expand them horizontally."

Exercises on Chapter XV

a. What is meant by *inductive inference*, and how is it related to *deduction*?

b. How are *observation* and *experiment* related to each other? Give examples to illustrate your answer.

c. What is meant by *analysis* and *synthesis*, and what is their function in the work of science?

d. What is an *hypothesis*, and what part does it play in scientific research?

e. What is a *barren hypothesis*, and how is it regarded in science?

f. What do you understand by *analogy*, and what is the chief function of analogy in science?

g. What is meant by *reasoning from analogy*, and why is there no *proof from analogy*? Give an example to illustrate your answer.

h. Explain how exactly the following passages illustrate what you have learned about the origin, nature, and verification of hypotheses.

(1) Lightning travels in a zigzag line, and so

does an electric spark; electricity sets things on fire, so does lightning; electricity melts metal, so does lightning. Animals can be killed by both, and both cause blindness. Electricity always finds its way along the best conductor, or the substance which carries it most easily, so does lightning; pointed bodies attract the electric spark, and in the same way lightning strikes spires and trees and mountain-tops. Is it not most likely that lightning is electricity passing from one cloud to another just as an electric spark passes from one substance to another?

If lightning is electricity, it must be possible, with the proper equipment, to draw this electricity to the earth. Accordingly, a kite was sent up during a thunderstorm, and a connection was thus established between the clouds and the earth. To the end of the string by which the kite was held there was tied a metal key. The string was then lengthened with some silk Since silk is a bad conductor of electricity, the electricity would be collected in the key, instead of escaping through the hand that held the silk. It was then found that if the key was touched with the finger, the usual effects of contact with electricity resulted.

(2) Cleomedes, who lived about the time of the Emperor Augustus, observed that a ring lying at the bottom of an empty vessel and just hidden from view by the side of

the vessel became visible when the vessel was filled with water. He thereupon suggested that the sun may already be below the horizon when we still see it at sunset.

(3) Were it a fact that the real interests of civilization are bound up with warfare, warfare would continue. But it is no more possible for nations to get rich by bombarding things with cannon and blowing its own customers and its own investments into smithereens than it is for a Wall Street financier to enrich himself by shooting Rockefeller or Morgan in the head.

(4) Sir Charles Lyell, by studying the fact that the river Ganges yearly conveys to the ocean as much earth as would form sixty of the great pyramids of Egypt, was enabled to infer that the ordinary slow causes now in operation upon the earth would account for the immense geological changes that have occurred, without having recourse to the less reasonable theory of sudden catastrophes.

(5) Woman is justly entitled to participate in the government of the State. For the government of the State is only a kind of national housekeeping. And all admit that woman has a genius for housekeeping.

Exercises on Chapter XVI

a. What is the general nature of *circumstantial evidence*, and what other kind of *evidence* is there? Give examples.

b. Why is the kind of reasoning exemplified by reasoning from circumstantial evidence called *systematic reasoning*? What other kind of *reasoning* is there? Illustrate your answer.

c. Give an example of reasoning from circumstantial evidence drawn (i) from the annals of crime, (ii) from the annals of history or of science.

d. How is systematic reasoning related to generalization?

Exercises on Chapter XVII

a. What is meant by *classification*, and how is scientific classification related to the orderly arrangement or grouping of things in museums and elsewhere?

b. In what sense is classification the oldest and simplest method of science?

c. What is the difference between a *natural classification* and an *artificial* one? Give an example of each.

d. What is meant by *description*, and what is the difference between a merely *qualitative* and a *quantitative* description? Give an example of each.

e. Describe the way in which statistical method is used as an aid to description, and give an example.

f. Explain the terms *arithmetical average* (or *mean*) *mode*, and *median*.

g. Two adjacent flower beds of equal size were prepared in the same way except that only one of them was manured with guano. On each bed was sown half the harvest from certain plants which had produced many flowers the year

before. During the summer one thousand flowers from each bed were examined in order to ascertain the number of petals per flower The result is given in the following table. Calculate (i) the *mean*, (ii) the *mode*, and (iii) the *median* of the number of petals per flower of each of the two kinds (manured and unmanured).

Number of Flowers.		Number of Petals per Flower.
With Manure.	Without Manure.	
140	120	5
150	150	6
170	250	7
210	210	8
140	120	9
90	100	10
40	30	11
30	10	12
20	10	13
10	0	14
1,000	1,000	—

h. De Vries carried out investigations on bulbous buttercups in order to ascertain the number of petals per plant. He examined 4,385 early plants (which flowered in July or August) and 1,130 late plants (which flowered in September). His results are given in the following table. Find (i) the *mean*, (ii) the *mode*, and (iii) the *median* of the number of flowers per buttercup

plant for each of the two groups of plants (early and late).

Number of Early Plants.	Number of Late Flowers	Number of Petals per Plant.
409	40	5
532	52	6
638	126	7
690	165	8
764	204	9
599	215	10
414	177	11
212	104	12
80	35	13
29	8	14
18	4	15
4,385	1,130	—

i. Calculate (i) the *average deviation*, (ii) the *median error*, and (iii) the *probable error* of the number of petals per plant, from the data supplied in the table given under *g*.

j. Calculate (i) the *average deviation*, (ii) the *median error*, and (iii) the *probable error* of the number of petals per flower, from the data in the table under *h*.

k. Give an account of *definition*, and give examples of its chief types.

l. Explain the terms *genus*, *species*, *differentia*, *proprium*, *accidens*, and illustrate your answer with suitable examples.

m. Define the following terms, indicating briefly in each case your mode of procedure:—*catalogue*,

dictionary, *cheque*, *society*, *state*, *federal state*, *imperialism*, *air*, *water*, *alcohol*, *republic*, *science*.

n. What is meant by *division*, and how is it related to *classification*?

o. Montesquieu divided Governments into Despotisms, Monarchies, and Republics. Lord Morley has criticized Montesquieu for employing two principles of division. Explain. Is it never permitted to use two principles of division?

p. The question whether or not a shovel is a spade was gravely discussed by half a dozen lawyers at Omaha before the president of the Board of the United States General Appraisers. If shovels are not spades, importers must pay a tariff of 20 per cent. on them. But if a shovel is a spade, it can enter the United States free of duty, as agricultural implements are exempted.

What point of logical interest does the above question raise?

Exercises on Chapter XVIII

a. What is a *comparative science*? Enumerate some of them.

b. What is meant by the *evolutionary* (or *genetic*) *method*?

c. Explain how some of the comparative sciences came to apply the evolutionary method.

d. How has it come about that the term *comparative method* is sometimes used as the equivalent of *evolutionary method*?

e. Why do some men of science object to the identification of the terms *comparative method* and *evolutionary method*?

f. Is there any one specific method that can appropriately be called the *comparative method* as distinguished from the *evolutionary method*? Illustrate your answer.

g. Why is the *genetic method* sometimes called the *historical method*? What objection is there to this usage?

h. Explain the difference between the terms *evolutionary hypothesis*, *evolutionary theory*, and *evolutionary method*.

i. What is meant by *working idea*, *principle*, *postulate*?

j. Give examples of the application of the evolutionary method from the fields of zoology and anthropology. (The student is advised to summarize for himself some examples from Darwin's *Origin of Species*, Frazer's *Golden Bough* and *Folklore of the Old Testament*, or similar works, including histories of art.)

k. How would you explain the remarkable feat of Agassiz which is reported in the following account?

> At a meeting of geologists in England, Agassiz was asked what kind of fish would be found in fossil form in a particular stratum of the earth's crust. He thought for a moment, and then made a sketch of the fish which he believed would live at the time this geological stratum was formed. He did not understand the cheer with which his drawing was received by the company, until someone brought forward an actual fossil specimen which had just been found, and showed that it agreed perfectly with the sketch which Agassiz had created from his

own knowledge of what characteristics a fish belonging to a particular stratum ought to possess.

Exercises on Chapter XIX

a. Explain (with examples) the meaning of ***uniformities of sequence*** and ***uniformities of co-existence***.

b. What is the relation between a ***rational connection*** and a ***causal connection***? Give examples.

c. Explain the terms ***positive instance*** and ***negative instance***.

d. Describe the nature of the ***method of difference***.

e. What different forms of the method of difference are there? Give an example of each.

f. What conclusions would you draw from the following accounts, and on which of the inductive methods do you base your conclusions?

(1) Wort, when it is prepared and stored in such a way that no dust can enter it, remains uncontaminated indefinitely. But when it is exposed to the air, under conditions which allow it to gather the dusty particles which float in the atmosphere, then it deteriorates.

(2) If the lungs be emptied as perfectly as possible and a handful of cotton-wool be placed against the mouth and nostrils, and you inhale through it, it will be found on expiring this air through a glass tube that it is free from such floating matter as is usually found when the air is inhaled directly.

(3) A freshwater crayfish with its antennules (or

small feelers) intact retreats from strong odours. But when these antennules are removed, the crayfish does not respond to strong odours.

(4) Two monkeys were made drunk, one with raw spirits, the other with matured spirits. The first became angry; it spat and it swore. The second was merely foolish and amiable in its intoxication. A week later the doses were reversed, and once more the monkey which had the raw whisky became quarrelsome, while the monkey which had the matured whisky was genial and good-humoured. [Why were the doses reversed?]

g. Explain and illustrate the fallacy of *non ceteris paribus*.

h. What is meant by the fallacy *post* (or *cum*) *hoc ergo propter hoc*? Give an example.

i. Describe the character of the *method of concomitant variations*.

j. In what respect are the methods of difference and of concomitant variations (i) like one another, (ii) different?

k. Give examples of (i) *direct concomitant variation*, (ii) *inverse concomitant variation*.

l. What conclusions would you draw from the following accounts, and on which inductive methods do you base your conclusions?

(1) If an active leaf be submerged in water contained in a glass vessel and exposed to the light, then bubbles may be seen coming from the surface of the leaf and rising through the water. (The water is

only a device by which the bubbles of gas may be seen.) If the leaf is very active, the bubbles are numerous. If the light is diminished gradually, the bubbles become fewer, and eventually cease altogether. If next the light is increased again gradually, the bubbles reappear, and become more and more numerous as the light increases.

(2) The great famine in Ireland began in 1845, and reached its climax in 1848. During these years agrarian crime increased rapidly, and in 1848 was more than three times as great as in 1845. After this time it decreased with the return of better crops, and in 1851 was only 50 per cent. more than in 1845.

m. Describe the *method of agreement*, and give an example of its application.

n. In what respect do the instances required for the *method of difference* differ from those which are sufficient for the *method of agreement*?

o. In what respect are the instances required for the *method of concomitant variations* (i) like, (ii) different from, those which suffice for the *method of agreement*?

p. What inferences would you draw from the following accounts, and on which inductive methods do you base your inferences?

(1) To-day a certain peculiar type of climate prevails wherever civilization is high. In the past the same type of climate seems to have prevailed wherever a great civilization arose.

(2) Observations made in a variety of cases have shown that whenever ether is administered to patients they breathe more deeply than before.

(3) Investigations in Denmark, Japan, Connecticut, Pennsylvania, New York, Maryland, the Carolines, and Georgia show that neither the winter nor the summer is the most favourable season for activity. Both physical and mental activity reach pronounced maxima in the spring and fall, with minima in midwinter and midsummer.

(4) All acids accelerate chemical reactions. Now acids vary from one another in all sorts of ways, except that they all contain ions of hydrogen.

(5) Hot springs are irregularly distributed in various countries throughout the world—in America, Tibet, Japan, Iceland, the Azores, the Pacific Islands, etc. It is found, however, that they practically always occur in regions which are, or have been, scenes of volcanic activity.

q. Describe the *method of residues*, and give an example of its application.

r. How is the *method of residues* related to the *method of difference*?

s. What inferences would you draw from the evidence contained in the following accounts, and on which inductive methods do you base your inferences?

(1) When air is confined with moistened iron filings in a closed vessel over water, the

iron filings rust, and the volume of air is diminished. Moreover, the iron filings, after rusting, gain as much in weight as the air loses.

(2) M. Arago, having suspended a magnetic needle by a silk thread, and set it in vibration, observed that it came much sooner to a state of rest when suspended over a plate of copper than when no such plate was under it. Now in both cases there were reasons why the needle should come at length to rest, namely, the resistance of the air and the want of perfect mobility in the silk thread. But the effect of those influences was exactly known from the observations made in the absence of the copper, but was not sufficient to account for the vibration ceasing as soon as it did.

(3) When J. J. Thomson investigated neon in a positive ray apparatus, he found that the parabola given on the photographic plate by the positive rays from neon was always accompanied by another parabola much less intense. The neon could not account for the less intense parabola.

t. Describe the character of the *joint method of agreement and difference*. Explain its relation to the *method of agreement* and to the *method of difference*. And give an example of its application.

u. What conclusions would you draw from the following accounts, and on which inductive methods do you base your conclusions?

(1) Cabbage leaves are much liked by worms; and it appears they can distinguish between different varieties. This must be due to differences in the taste of the leaves or to differences in their texture. Pieces of the leaves of cabbage, turnip, horse-radish and onion were left on the pots during twenty-two days and were all attacked and had to be renewed, but during the whole of this time leaves of an Artemisia, and of the culinary sage, thyme, and mint, mingled with the above leaves, were quite neglected, excepting those of the mint, which were occasionally and very slightly nibbled. These latter four kinds of leaves do not differ in texture (from the others) in a manner which could make them disagreeable to worms; they all have a strong taste, but so have the four first-mentioned kinds of leaves.

(2) Darwin observed that in that part of the country where he lived, clover was abundant in the fields which were situated near villages, while the outlying fields were almost destitute of it. Now the outlying fields harboured plenty of mice, whereas the cats from the villages destroyed the mice in the fields near by.

(3) Galen noticed on various occasions that the pulse of one of his female patients quickened whenever the name of Pylades the dancer was mentioned. But there was no marked quickening of the pulse when

the name of Morphus or of any other professional dancer was mentioned.

v. What is meant by *relevant antecedent* in the formulation of the inductive methods? And how can the relevant be distinguished from the irrelevant?

Exercises on Chapter XX

a. What is meant by the *method of simple enumeration*, and of what scientific value is it?

b. What is the most important difference between *statistical method* and the *method of simple enumeration*?

c. Under what circumstances can statistical methods be employed though the ordinary inductive methods cannot?

d. What are the principal steps in the application of statistical methods?

e. What is meant by a *contingency table*? Give an example.

f. Explain in a very general and simple way what is meant by (i) *association of attributes*, (ii) *correlation of variables*, and (iii) *frequency*.

g. Having regard to the data supplied in the table given under *g* in the exercises on Chapter XVII, what inference would you draw about the influence of the guano manure on the flowers concerned? Justify your conclusion.

h. Taking into account the data furnished in the table under *h* in the exercises on Chapter XVII, what conclusion would you draw about the effect of the season on the plants in question? Justify your answer.

i. Explain how you would deal with the question raised in the following passage if you could get the necessary data:—

> The old man who has been bathing in the Serpentine every morning for forty years says, "Look at me." I say to him, "Yes, but where are the others?"

j. What inference would you draw, and why, from the information contained in the following statement?

> In 1892, that is, before the suffrage was given to the women of Australia and New Zealand, the infant mortality rates per 1,000 births were: Australia, 106; New Zealand, 89. During the years following the granting of woman's suffrage there was a reduction in the infant mortality rate, which in 1912 was only 72 per 1,000 births for Australia, and 51 for New Zealand. In England and Wales, where the suffrage was not yet given to the women, the infant mortality rate per 1,000 births was 148 in 1892, and only 95 in 1912.

k. Describe the chief precautions that have to be taken in interpreting statistical data.

l. When do statistical records cease to be of scientific importance, and why?

Exercises on Chapter XXI

a. Explain the general character of the *deductive-inductive method*, and its principal forms.

b. What are the main uses of the *deductive-inductive method*? Give an example of each of them.

c. Describe the different senses in which the term

theory is used, comparing it with the terms *hypothesis* and *law*.

d. What inferences would you draw from the data contained in the following accounts, and on which inductive methods do you base your conclusions?

(1) By mathematical reasoning Cavendish arrived at the conclusion that if the force between two electric charges varies inversely as the square of the distance between them (and in no other case), then electricity communicated to a body must collect wholly on its surface, so that the interior will be uncharged.

He placed one metal sphere inside another metal sphere in such a way that each was insulated independently, and then charged the outer sphere and connected it momentarily by a wire with the inner sphere. After breaking the connection, he tested the inner sphere with an electroscope, but could find no charge upon it.

(2) Some people object to the freedom allowed to Parliament and the Press to make criticisms and disclosures that may lower the rulers in the eyes of the people. Well, there are no complaints against the Government in Turkey—no motions in Parliament, no *Morning Chronicles*, and no *Edinburgh Reviews*. Yet of all countries in the world, it is in Turkey that revolts and revolutions are most frequent.

The greater the quantity of respect a

man receives, independently of good conduct, the less good is his behaviour likely to be. It is the interest, therefore, of the public, in the case of each, to see that the respect paid to him should, as completely as possible, depend upon the goodness of his behaviour in the execution of his trust, and this is only possible by permitting free criticism.

(3) Pasteur had shown that the microbe of anthrax does not develop at a temperature of 44° C. The temperature of birds being 41–42°, it was argued that fowls would be immune from anthrax, as their blood is warm, and their vitality would help them to bridge over the small gap between 41° and 44° C.

A hen was accordingly inoculated with anthrax blood and placed with its feet in water at 25°. The blood of the hen got cooled to 37°. After 24 hours it was dead, and its blood was full of anthrax bacteria. Another hen similarly inoculated and cooled until it was in a high state of fever was then taken out of the water, wrapped in cotton wool, and placed in an oven at 35°. Its strength gradually returned, and in a few hours it was quite well again.

(4) According to Renan, the fear of conquest and the consequent preparation for war are a necessary spur to human progress. But consider the evidence. In Russia, in England, and in other countries where the

armament competition is acute, there are myriads of people who live below the bread-line. In America, on the other hand, conditions are far better. Even European countries like Norway, Sweden, Denmark, and Switzerland, which during the past fifty years have not been under the fear of war, have also made great progress. The fear of conquest has really retarded the progress of the Great Powers. Their armament competition has eaten up two-thirds of their revenues, to the detriment of education and social reform.

(5) At the time of the American Revolution many Loyalists left their homes in the Southern States and went to the Bahamas. Other colonists also went there from Great Britain. Now after several generations the descendants of those Loyalists show a larger proportion of degenerates than can be found in any other Anglo-Saxon community. The descendants of similar Loyalists in Canada are among its strongest elements; in the Bahamas they are scarcely ahead of the average negro.

And naturally so. For they lose in both physical and mental energy. This leads to carelessness in matters of sanitation and food, and thus gives greater scope to the diseases which under any circumstances would find an easy prey in their weakened bodies. The combination of mental inertia and physical weakness makes it difficult to overcome the difficulties arising from

isolation, from natural disasters, or from the presence of an inferior race, and this in turn leads to ignorance, prejudice, and idleness.

Additional Exercises on Chapters XV–XXI

In the following passages various investigations are summarized very briefly. Just enough is given in each passage to indicate the general character of the procedure and of the methods employed. What is required is a description of this procedure and of these methods in the light of the logic of induction, or the study of scientific method. The general mode of treatment to be followed in the analysis of these passages is, perhaps, best indicated in the following list of questions, remembering always that not all the considerations here enumerated will be called for in all cases.

(*a*) What is the *problem* considered in the passage?
(*b*) What *hypothesis*, or hypotheses, does it propound? And how was the hypothesis suggested?
(*c*) What *evidence* is adduced *pro* or *con*?
(*d*) What *method*, or methods, does the evidence follow?
(*e*) Is there adequate *verification*?
(*f*) What *fallacy* is committed or imputed?

The first four exercises are worked out here as examples.

1. The phenomena of shells found in rocks, at a great height above the sea, has been explained in various ways. Some ascribed it to a plastic power in the soil; some to the influence of the heavenly bodies; some to fermentation; some to the passage of pilgrims

with their scallops; some to the life and death of real molluscs at the bottom of the sea, and a subsequent alteration of the relative level of the land and sea in that region. Now as regards the plastic power of the soil and celestial influence, the very existence of such powers is unknown. Fermentation is a real or true cause, but it has never been observed to produce shells Casual transport by pilgrims is a true cause, and might account for a few shells, but not for so many. On the other hand, it is a usual thing for a shell-fish to die at the bottom of the sea and leave its shell in the mud; and the elevation of the bottom of the sea to dry land has been witnessed often.

[*Problem* : To account for the presence of shells found at a great height above the sea.

Hypotheses : There are five rival hypotheses, each suggesting a different cause or agent. Now hypotheses with *veræ causæ* (true causes, that is, causes otherwise known to exist) are preferred to those with purely hypothetical causes. This disposes of the first two hypotheses. Again, of *veræ causæ* preference is given to those which are known not only to exist, but also to act in the kind of way required. This disposes of the third hypothesis. The fourth hypothesis cannot account for *all* the shells, and would therefore still leave a residual problem. The last hypothesis is the only one that would account for all the facts, and it is supported by close analogies of observed facts.]

2. It was a general belief at St. Kilda that the arrival of a ship gave all the inhabitants colds. Dr. C. took pains to ascertain the fact and to explain it as the effect of effluvia arising from human bodies; it

was discovered, however, that the situation of St. Kilda renders a north-east wind absolutely necessary before a ship can make the landing.

[*Problem* : What causes these epidemics of colds?

Current Hypothesis: The arrival of ships.

Evidence : Probably many occasions on which the arrival of a ship was followed by such an epidemic. *Method of Difference* as regards each occasion, on the (erroneous) assumption that no other relevant change had occurred; the *Joint Method of Agreement and Difference* if many occasions (positive and negative) are compared.

Objection : No apparent connection between the alleged cause and the effect.

Auxiliary Hypothesis (to meet the objection, by inserting a connecting link): The effluvia given off by the new arrivals causes the epidemics. But this alleged agency is purely hypothetical; it is not a *vera causa*, that is to say, there is no other evidence of its existence.

New Hypothesis: The north-east wind is the real cause of the colds. This is a *vera causa* of colds, and would therefore be more acceptable than the hypothetical effluvia.

Evidence : Same methods as above, only substituting north-east wind for the arrival of a ship, which may be regarded as an irrelevant circumstance. The evidence might also have included the occurrence of epidemics after north-east winds even when no ships arrived. Such evidence would be even more clinching; but we are not told.]

3. The effect of green feed on the colour of the yolks of eggs has been studied recently by Professor

Wheeler of New York. Four lots of hens were fed alike, except that no hay or green feed was given to one lot, while the other three lots had varying amounts of clover hay alternating with green alfalfa. The depth of colour of the yolk varied in the different lots, and, roughly speaking, was directly proportional to the amount of the clover and alfalfa on which the laying hens were fed. Apparently the colouring matter present in the green feed affects the yellow colouring matter of the yolks of eggs.

[*Problem*: What determines the colour of the yolk of eggs?

Hypothesis: The amount of green feed given to the hens.

Verification: The lot of hens which received no green feed laid eggs the yolks of which had no colour to speak of. The three lots which had green feed all laid eggs with coloured yolks. Group form of the *Method of Difference* (the three lots being regarded as one positive group). Again, among the three positive lots of hens the depth of the colour of the yolks varied concomitantly with the amount of green feed given to the hens that laid the eggs. *Method of Concomitant Variations.*]

4. Pliny rejected the belief in nativities (i.e. the determination of a man's destiny by the star under which he was born) on the ground that masters and slaves, kings and beggars are frequently born at the same time.

[*Problem*: The relation between human destiny and the heavenly bodies.

Old Hypothesis: A man's destiny is determined by the star under which he is born.

Evidence for: Not stated. Probably observation of a few coincidences according to the *Method of Simple Enumeration*, and the ignoring of exceptions.

Evidence against: If the hypothesis were true, then people born under the same star should have similar destinies; but they have not. *Method of Agreement with negative result.**

*The student is advised to make himself familiar with the conception of a *negative result* (not to be confused with a *negative instance*!). When an inductive method is applied to confirm or to test an hypothesis, and the result does not confirm the hypothesis, or even disproves it, then the result is said to be negative—it is not what it should have been if the hypothesis had been true. Beginners usually find it difficult to realize that the method has the same character even when the result is negative—just as a train journey remains a train journey even when our friend happens to be away when we reach our destination.]

5. Goldscheider had his arm suspended in a special frame and moved about by an assistant. His muscles, therefore, had no share in these movements. Yet he could distinguish as small an angle of movement of this arm as when his own muscles moved and supported it. He concluded that muscular sensations play no important rôle in our consciousness of the movements of our limbs.

6. It has been found that linnets when shut up and educated with singing larks—the skylark, woodlark, or titlark—will adhere entirely to the songs of those larks, instead of the natural song of the linnets. We

may infer, therefore, that birds learn to sing by imitation, and that their songs are no more innate than language is in man.

7. If we breathe on a cold metal or stone, moisture condenses on it. The same phenomenon appears on a glass when ice-water is poured into it, and on the inside of windows when the air outside gets colder suddenly. We may therefore conclude that condensation of moisture on a surface is due to its being colder than the surrounding air.

8. Pasteur filled part of a bottle with wine, and sealed the bottle hermetically. Presently the wine changed into vinegar. Pasteur then submerged the bottle well under water and then withdrew the cork. The water rushed into the bottle and filled just one-fifth of the space originally occupied by air. Now, air is composed of one part of oxygen to four parts of nitrogen. Moreover, the gas left in the bottle had all the properties of nitrogen. Pasteur therefore concluded that during the process of acetification oxygen is taken from the air.

9. When we put any limb in motion the seat of the exertion appears to be the limb, whereas it is demonstrably no such thing, but either in the brain or in the spinal marrow; the proof of which is, that if a little fibre, called a nerve, which forms a communication between the limb and the brain, or spine, be divided in any part of its course, however we may make the effort, the limb will not move.

10. Dorfmeister has shown that the same chrysalis, according as it was submitted to cold or heat, gave rise to very different butterflies, which had long been regarded as independent species, *Vanessa levana* and

V. prorsa; an intermediate temperature produces an intermediate form.

11. According to the observation of Ptolemy (second century A.D.), the position of the sun among the stars, at the time of its greatest distance from the earth, was in longitude 65°. According to the observation of El-batani, the Arabian astronomer (*c.* 900), it was in longitude 82°. The difference is too great to be accounted for by inaccuracy of measurement. The solar system is probably itself moving through space.

12. Sachs found that when light was excluded from a plant then, although all other conditions remained the same, no starch was formed; but when the plant was exposed to light again, then there was a renewed formation of starch. Similarly, when certain portions of the leaves of an illuminated plant were covered with black paper, then no starch was formed in those portions. Sachs concluded that starch is formed in plants by the decomposition of carbon-dioxide gas in chlorophyl under the influence of light.

13. Schwabe discovered that sun-spots reached a maximum once in approximately ten years. Lamont found that magnetic storms showed a periodicity of about ten years Sabine discovered independently that magnetic disturbances reached a maximum of violence and frequency at intervals of about ten years. He noted, moreover the coincidence between the period of magnetic storms and that of sun-spots; and showed that, according to the available data, the two cycles of change agree in duration and phase, maximum corresponding to maximum, and minimum to minimum. He concluded that there was some connection between them, though he could not explain the nature of the connection.

14. The inquiry into the cause of sound had led to conclusions respecting its mode of propagation, from which its velocity could be precisely calculated. The calculations were made; but when compared with actual observations, although the agreement was sufficient to confirm the general correctness of the suggested mode of propagation, yet the velocity was rather greater than that demanded by the theory. Eventually, however, Laplace struck on the happy idea that the excess velocity may be due to the heat developed in the act of condensation which takes place at every vibration by which sound is conveyed. The matter was subjected to exact calculation, and the result was at once a complete explanation of the phenomenon under consideration and a striking confirmation of the general law of the development of heat by compression, under circumstances beyond artificial imitation.

15. The process of nitrate production (nitrification) was formerly supposed to be entirely chemical. But in 1877, S. and M. showed that it was brought about by bacteria. A stream of sewage was made to trickle slowly down a column of sand and limestone. For the first twenty days the ammonia in the sewage remained unaltered, then it began to change into nitrate, and finally the issuing liquid contained no ammonia but only nitrate. S. and M. contended that if the process of nitrification was purely chemical the delay of twenty days before the ammonia was transformed into nitrate was inexplicable, but if it was bacterial then time would be required for these organisms to grow. As a further test some chloroform vapour was added. The nitrification ceased. But it commenced again when the chloroform was removed, and turbid extract of fresh soil was added.

16. In 1861 there died at the Bicêtre a patient who for twenty years had been without the power of speech, apparently through loss of memory of words. An autopsy revealed that a certain convolution of the left frontal lobe of his cerebrum had been totally destroyed by disease, the remainder of the brain being intact. Broca held that this case pointed strongly to a localization of the memory of words in a definite area of the brain.

17. During the Spanish-American War the American troops suffered great losses from yellow-fever. A commission was appointed to investigate the causes. It was thought likely that yellow-fever, like malaria, was spread by mosquitoes which had bitten patients suffering from the fever. Dr. L. accordingly allowed himself to be bitten by such a mosquito. He contracted the disease, and died within a few days. Next, three volunteers slept for twenty nights in a small, ill-ventilated room screened from mosquitoes, but containing furniture and clothing which had been in contact with yellow-fever patients, some of whom had died of the disease. None of the volunteers contracted the disease. Then a similar room was divided by a wire screen, and mosquitoes which had bitten yellow-fever patients were admitted on one side only of the screen. One of the volunteers entered this section, and allowed the mosquitoes to bite him. He had an attack of yellow-fever. Two volunteers who stayed on the other side of the screen, and were thus protected from mosquito bites, remained in perfect health.

18. It has been observed that the simpler the type of an animal's nervous system the fewer and more mechanical are the activities of which the animal is capable; while, on the other hand, the more elaborate

the nervous system the more complex and adaptable are its reactions. And, since intelligence generally shows itself by great adaptability to surroundings, it would appear that intelligence depends on the nervous system.

19. It used to be supposed that all acids contain oxygen. Now if this assumption were true the combination of ammonia (which does not contain oxygen) with hydrochloric acid should give as one of its products water, which would contain the oxygen previously contained in the hydrochloric acid. But when ammonia and hydrochloric acid were actually combined it was found that only a slight dew was formed, which could be accounted for by the unavoidable imperfections in the process of mixing the ammonia with the acid. It was therefore concluded that hydrochloric acid does not contain oxygen.

20. It was long known that light is a detriment to the preservation of milk. But until recently it was not known which of the rays did the mischief. Dr. P. put sterilized and unsterilized milk in uncoloured glass bottles, in red glass bottles, in orange-coloured glass bottles, and in glass bottles of the other colours of the spectrum. He then placed all the bottles in the light for a whole day. It was found, at the end of the day, that both kinds of milk in the red glass bottles were fresh, even the unsterilized milk being good still for many hours. But the milk in all the other bottles had "turned" more or less; the milk in the bottles having the colours of the violet side of the spectrum had "turned" most of all. Red rays, therefore, appear to be beneficial to the preservation of milk, the other rays being more or less beneficial, or more or less injurious, according as they are nearer to the red or nearer to the violet end of the spectrum.

21. In Glasgow, the general death-rate is twice as high in one-room apartments as it is in houses of four rooms or more. Even in three-room houses it is 25 per cent. higher. In Edinburgh, when certain slum-areas were demolished, the general death-rate fell from 45 to 15 per 1,000 of the population. The following table shows the weight and height of boys of 5, 9, and 11 years from homes of 1 2, 3, and 4 or more rooms:—

Number of Rooms.	Weight in Pounds.			Height in Inches		
	5 Years.	9 Years.	11 Years.	5 Years.	9 Years.	11 Years.
1	37·2	51·4	60·0	39·0	46·5	50·1
2	38·6	53·1	62·2	39·9	47·6	50·9
3	39·5	51·8	64·5	40·7	48·2	51·7
4	40·1	56·6	66·2	41·4	48·9	52·4

The worse the housing conditions, the punier the children.

22. When the law of gravitation is taken for granted, and applied to the actual conditions of our own planet, one of the consequences to which it leads is, that the earth, instead of being an exact sphere, must be flattened in the direction of its polar diameter, the one diameter being about thirty miles shorter than the other. Investigation showed this conclusion to be true in fact.

23. Dogs are liable to a fatal disease called rabies, which they transmit by their bite. When communicated to man this disease is known as hydrophobia. The specific virus of the disease is found in the saliva and salivary glands of the infected animal (hence the

transmission by a bite), but also in the spinal cord of the infected animal. Pasteur showed that inoculations with an emulsion of such a spinal cord reproduced the disease in dogs and rabbits. He also discovered methods of preparing such emulsions of varying intensity and virulence. He inoculated some dogs with emulsions of increasing strength. The dogs treated in this way survived when affected by strong virus, which proved fatal in other dogs.

24. In 1909 the Midland Agricultural and Dairy College tested the value of applying certain fertilizers to pasture land used for dairy cows. Two plots of pasture land, *A* and *B*, each four acres in area, were fenced off, and both plots were dressed with 10 cwt. of ground lime per acre. Plot *A* received nothing else. Plot *B* received an additional dressing of 4 cwt. superphosphate and 1½ cwt. sulphate of potash per acre. Neither plot received any further dressing during 1909-1912. Two lots of cows were drafted on to the plots each year early in May, and were kept there continually as long as there was any grass, no other food being given. At the end of each fortnight the two lots were changed over, those on plot *A* going on to plot *B*, and *vice versa*. The net increase of profit on milk for plot *B* was an average of over £30 per annum for 1909-1912. In 1913 the test was repeated, the treatment of the plots being reversed. And the result is now similarly favourable to plot *A*.

25. H. and W. were studying the effect of nitrates on plant growth, and found that the amount of growth of cereals like barley and oats increased as the nitrate supply increased, and was, in fact, directly proportional to the amount of nitrate. In the case of peas and allied plants, however, no sort of proportionality

could be traced. The plants did sometimes as well (or better) without nitrate as with it, but sometimes failed altogether. Chemical analysis showed that the quantity of nitrogen present in the cereal crops was just about the same as that applied to the soil, while the quantity present in those peas which made any growth was much greater. These peas must, therefore, have got some of their nitrogen from the air. But why had not all the peas done so? H. and W. suggested that bacteria might be the active agents here, as they knew that the nodules (the little swellings on the roots of the peas) contained bacteria, and also that some bacteria could take in gaseous nitrogen and use it. To test the matter, peas were sown in sterilized sand (i.e. sand baked so as to kill all living organisms it contained) holding mineral food, but no nitrogenous food. These made little growth and developed no nodules. Other peas were sown in similar sand, but with the addition of a water extract containing organic matter. These made excellent growth and had many nodules. But if the water extract was first boiled, it had no effect in increasing growth. This showed that peas can associate with certain bacteria as so to draw on the stores of nitrogen in the air.

26. In Davy's experiments with the decomposition of water by galvanism it was found that, besides oxygen and hydrogen, an acid and an alkali were developed at the two opposite poles of the machine. Davy conjectured that the glass containing the water might suffer partial decomposition, or some foreign matter might be mingled with the water, and the acid and alkali be disengaged from it. He substituted gold vessels for glass; but it made no difference. Evidently the glass was not the cause. He next employed only

distilled water. There was a marked reduction in the quantity of acid and alkali evolved; but they were still there. He next suspected the perspiration from the hands touching the instruments, as it would contain common salt, which would decompose into an acid and an alkali under the action of electricity. After carefully avoiding manual contact, only slight traces of acid and alkali appeared, such as might be traced to impurities from the atmosphere, decomposed by contact with the apparatus. He next put the machine under an exhausted receiver. It evolved no acid or alkali.

27. It was well known that a falling body moves faster and faster as it travels through space. Galilei set himself to examine how this increase of velocity occurs. If he had worked by the Baconian method, he would have made endless experiments on falling bodies, till relations forced themselves on his notice. He did nothing of the kind. He thought over the facts, and made a guess at a possible law. He surmised that the speed of a falling body might be proportional to the *distance* fallen through. He reasoned out the consequences of this supposition, and found that they were self-contradictory. Accordingly, he tried a different supposition, namely, that the speed varied as the *time* of the fall. Here the consequences deduced were consistent; the suggestion was worth testing. Owing to the lack of suitable apparatus he could not put his suggestion to a direct test. He therefore took one of its consequences (which he had obtained by mathematical reasoning)—the consequence, namely, that the distance fallen should be proportional to the square of the time. He made a falling body run down an inclined plane, and measured (by means of a new kind

of water-clock) the times taken to run over marked distances on the plane. Small divergences in individual cases appeared; but, on the average, his results showed that the distances traversed were proportional to the squares of the times of fall. No other supposition could account for these results. Galilei accordingly considered his second surmise to be accurate.

28. Poulton and Sanders experimented with 600 pupæ of the tortoise-shell butterfly, in order to test the theory of protective coloration. The pupæ were artificially attached to nettles, tree-trunks, fences, walls, and to the ground, some at Oxford, some at St. Helens in the Isle of Wight. In the course of a month 93 per cent. of the pupæ at Oxford were killed, chiefly by small birds, while at St. Helens 68 per cent. perished. At Oxford only four pupæ, fastened to nettles, emerged; all the rest perished. At St. Helens the eliminations were as follows: on fences, where the pupæ were conspicuous, 92 per cent.; on bark, 66 per cent.; on walls, 54 per cent.; and among nettles, 57 per cent. The experiments showed very clearly that the colour and character of the surface on which the pupæ rests—and thus its own conspicuousness—are of the greatest importance.

29. Nations did not originate by simple natural selection, as wild varieties of animals no doubt arise in nature. You could not show that the natural obstacles opposing human life much differed between Sparta and Athens or indeed between Rome and Athens; and yet Spartans, Athenians, and Romans differ essentially. Old writers fancied that the direct effect of climate, and the sum total of physical conditions varied man from man, and changed race to race. But experience refutes this. The English immi-

grant lives in the same climate as the Australian or Tasmanian, but he has not become like those races. The Papuan and the Malay live now, and have lived for ages, side by side in the same tropical regions, with every sort of diversity. Even in animals the direct efficacy of physical conditions is overrated. Borneo and New Guinea, as alike physically as two distinct countries can be, are zoologically as wide as the poles asunder; while Australia, with its dry winds, its open plains, its stony deserts, and its temperate climate, yet produces birds and quadrupeds which are closely related to those inhabiting the hot, damp, luxuriant forests which everywhere clothe the plains and mountains of New Guinea. That is, we have like living things in the most dissimilar situations, and unlike living things in the most similar ones. Nor can we doubt that we find like men in contrasted places, and unlike men in resembling places Climate clearly is *not* the force which makes nations, for it does not always make them, and they are often made without it.

30. The most surprising thing about the Mayas is that they developed their high civilization in what are now the hot, damp, malarial lowlands where agriculture is impossible. A hundred miles away far more favourable conditions now prevail. In the past these more favourable localities were occupied by people closely akin to the Mayas, yet civilization there never rose to any great height. In explanation of these peculiar conditions three possibilities suggest themselves: (1) We may suppose that the Mayas were able to carry on agriculture under conditions with which no modern people can cope; that they chose the worst place even though far better places lay close at hand and were occupied by allied peoples few in numbers and back-

ward in civilization; that for a thousand years they were able to preserve their energy under the most debilitating climatic conditions, immune to the many fevers which to-day weaken the dwellers there. (2) Or we may suppose that in the time of the Mayas tropical diseases were less harmful than they are now. (3) Lastly, we may suppose that the climate has changed—the dry conditions which prevail a little farther north may have prevailed in the Maya region when the Mayas attained eminence. That climates do sometimes change is evidenced by conditions in Palestine. In Palestine the rainy zone once extended at least fifty miles south of its present limit, but the zone of cyclonic storms has at certain periods suffered a shift equatorward. Now if the Maya region has undergone a corresponding climatic change the explanation would be simple. A longer dry season would diminish the amount of vegetation and cause scrub to take the place of dense forest. Agriculture would then be comparatively easy, and fevers would greatly diminish.

Exercises on Chapter XXII

a. What is meant by *probability*?

b. What is usually meant when an *event* is described as *probable*?

c. What is the difference between *calculable* and *incalculable probability*? Give an example of each.

d. Briefly explain, and illustrate, the relation between the *deductive* (or *a priori*) and the *inductive* (or *a posteriori*) calculation of probability.

e. Enumerate and explain the conditions under which it is possible to calculate probabilities *a priori*.

f. Explain the relation between *probability* and *odds*,

and show how a knowledge of either enables one to calculate the other.

g. Explain the way in which it is sometimes attempted to base *induction by simple enumeration* on the calculus of probability.

h. Why is a *theory* usually less probable than any one of the *laws* which it systematizes?

i. Why does an hypothesis become more and more improbable in proportion to the number of auxiliary hypotheses which it needs in order to account for all known relevant facts?

j. It is said that the knowledge of the probability of an event gives no definite information about the next or any future occurrence of an event of that kind. Explain this view, and show the real nature and value of such knowledge.

k. Explain (with special reference, say, to fire insurance) how calculations of probability may save us from some of the consequences of the irregularities of details by invoking the regularity of the average of the class.

l. Compare the way in which you would calculate the chances that a normal die when thrown will show a particular number, with the way in which you would calculate the chances in favour of a man's recovery from a certain illness. Is the underlying principle the same in the two cases? If not, explain the difference.

m. A party of twelve persons, including A and B, take their seats at random at a round table. What is the probability that A and B will sit next to each other?

n. Out of a thousand children who are ten years old, seventy die before reaching the age of twenty-

five. Let A, B, and C stand for three children, each of them ten years old. Calculate the probability (i) that B will reach the age of twenty-five, (ii) that at least one of the three will live to the age of twenty-five, (iii) that both A and B will reach the age of twenty-five.

o. Four boats, P, Q, R, and S, ply regularly between two seaports, X and Y. Calculate the probability that a passenger, who has gone from X to Y and back, made (i) his outward passage in either P or S, (ii) both passages in R, (iii) both passages in the same boat, (iv) at least one passage in Q.

p. In my pocket I have two sovereigns, three single shillings, and six halfpennies, and I take out two of the coins at random. Calculate the chances that the two coins are (i) the two sovereigns, (ii) a shilling and a halfpenny.

q. Five hens of the same strain (A, B, C, D, E) lay four eggs on a certain day. Calculate the chance (i) that A laid one of the eggs, (ii) that both B and C were among the layers, (iii) that a particular egg was laid by D.

r. A and B arrange to meet at the entrance of an exhibition which, without their knowing it, actually has four different entrances, W, X, Y, Z. Calculate the probability (i) that A and B will meet at the same entrance; (ii) that they will meet at entrance Z; (iii) that A will arrive at entrance X and B at Y.

s. X, Y, and Z are known to have been in a train-wreck in which a third of the passengers were injured. Supposing that no other information is available, calculate the probability (i) that

X and Y, (ii) that X and Y and Z, (iii) that X or Y, (iv) that X or Y or Z, have *escaped* injury.

t. A, B, and C are respectively ten, twelve, and fourteen years of age. Supposing that of children who are ten, twelve, and fourteen years old, 7, 6, and 5 per cent. respectively die before attaining the age of twenty-five, calculate the probabilities that (i) A and B, (ii) A or B, (iii) A or B or C will survive till the age of twenty-five.

u. Unaware that his watch has stopped, Smith looks at it once in the course of twelve hours. Assuming that it takes him a whole minute to tell the time, calculate the probability that he will see the correct time.

v. What is the probability that the first two persons you meet were born on (i) the same day in the week, (ii) a Sunday?

w. Two children who can only count up to ten are asked to think of any number they like. What is the probability that they will both think of the same number?

x. A, B, and C are known to have been respectively 1st, 2nd, and 3rd class passengers on a boat which was wrecked with a loss of 30 per cent. of the 1st class passengers, 50 per cent. of the 2nd class passengers, and 70 per cent. of the 3rd class passengers. Calculate the chances of the safety of (i) A and B, (ii) A or B, (iii) A or B or C.

y. A, B, and C are known to have been passengers on a boat which was torpedoed with a loss of 80 per cent. of the 1st class passengers, 60 per cent. of the 2nd class passengers, and 70 per

cent. of the 3rd class passengers. Supposing it be known that A travelled 1st class and B 2nd class, and that no other information is available for the time being, calculate the probability that (i) A and B are saved, (ii) C is saved, (iii) A *or* B is saved, (iv) one of the three at least is saved.

x. Why is it better to take the mean of a number of observations than to trust one observation, however carefully made?

Exercises on Chapter XXIII

a. What is meant by *order in nature*?

b. On what grounds is it assumed that there is order in nature?

c. How far is science, as such, committed to the assumption that there is order in nature?

d. What significance is to be attached to the occurrence of apparent deviations from uniformities among natural phenomena?

e. What is meant by the *uniformity of nature*?

f. What is a *natural law*? In what different ways may natural laws be distinguished from each other? Give examples.

g. Explain the terms *cause* and *reason*, and their relation to one another.

h. Explain and illustrate the meaning of the terms *condition*, *positive condition*, and *negative condition*. How are *conditions* related to *causes*?

i. How is a *causal sequence* related to other kinds of sequence?

j. Discuss the tendency to replace the conception of *causality* by that of *law*, or to maintain that

the business of science is to ascertain the *correlations* of phenomena and not their *causal connections*.

k. Give an account of the *principle of fair samples* and its place in the domain of science.

Exercises on Chapter XXIV

a. Consider the view that science is not concerned with *explanation*, but only with *description*.

b. What do you understand by *description*, and how would you distinguish it from *explanation*? Give examples.

c. How would you define the term *fact*, and how would you distinguish between a *fact* and its *interpretation*? Give examples.

d. Enumerate the chief *types of explanation*, and give a brief account and an example of each of them.

e. What is meant by an *empirical law*, and how is it distinguished from other natural laws? Give examples.

f. In what way is the induction of *theories* different from the induction of *natural laws*, and how does this difference affect the comparative probability of the two?

g. What do you consider to be the logical justification of induction?

h. On what grounds has the validity of science been challenged, and in what way would you defend it?

Exercises on Chapter XXV

a. In what way is *logical reasoning* different from the mere play of fancy?

b. Compare the ***materialistic*** with the ***idealistic*** accounts of reasoning, and state the view which seems to you the least unsatisfactory.

c. Explain the meaning of the term ***particular***, and consider the place of the particular in reasoning.

d. Examine the view that inference is always from particular cases to particular cases.

e. Examine the view that inference always involves a universal.

f. Formulate the principle of ***uniformity of reasons***, and explain in what way it may be regarded as the ultimate assumption of all inference.

g. Is it permissible to describe all inductive inference as essentially deductive reasoning having the principle of uniformity of reasons (or some other principle) for major premise? Give your reasons.

h. In what way may the cultivation of reason be regarded as a potent aid to human harmony?

SELECT LIST OF BOOKS

I. General Treatises—

B. Bosanquet: *Logic* (2 vols., 2nd ed.).

W. E. Johnson: *Logic* (3 vols., 4th in preparation).

J. S. Mill: *System of Logic*.

C. Sigwart: *Logic* (2 vols.).

A. Wolf: *Exercises in Logic* and *Key to Exercises in Logic* (Allen & Unwin).

II. Formal Logic—

J. N. Keynes: *Formal Logic* (4th or later edition).

A. T. Shearman: *The Scope of Formal Logic* and *The Development of Symbolic Logic*.

A. Wolf: *Studies in Logic* (Camb. Univ. Press).

L. Couturat: *The Algebra of Logic*.

B. Russell: *Introduction to Mathematical Philosophy* (Allen & Unwin).

III. Inductive Logic—

W. S. Jevons: *Principles of Science*.

J. A. Venn: *Empirical Logic*.

A. D. Ritchie: *Scientific Method*.

F. W. Westaway: *Scientific Method*.

A. Wolf: *Principles of Science* (in preparation).

IV Special Topics—

J. M. Keynes: *A Treatise on Probability*.

A. L. Bowley: *Elements of Statistics*.

D. C. Jones: *A First Course in Statistics*, Part I.

G. U. Yule: *An Introduction to the Theory of Statistics*.

J. N. Keynes: *The Scope and Method of Political Economy*.

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